Physics

SOLUTIONS

Content

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IIT JEE | MEDICAL | FOUNDATION

Units Dimensions and Vectors

1.(A) By substituting dimension of each quantity in R.H.S. of option

(a) $\left[\frac{mg}{\eta r}\right]=\left[\frac{M \times LT^{-2}}{ML^{-1}T^{-1} \times L}\right]=[LT^{-1}]$

This option gives the dimension of velocity.

2.(A) By substituting the dimension of given quantities $[ML^{-1}T^{2}]^{x}\ [MT^{-2}]^{y}[LT^{-1}]^{2}=[MLT]^{0}$

By comparing the power of *M*, *L*, *T* in both sides.

$x+y=0$ . . . .(i)

$-x+z=0$ . . . . (ii)

$-2x-2y-z=0$ . . . . .(iii)

The only values of x, y, z satisfying (i), (ii), and (iii) corresponds to (b)

3.(C) In this question, density should be reported to two significant figures.

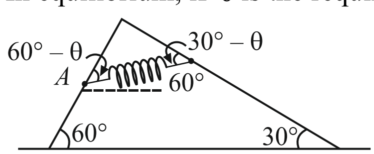
$$\text{Density}=\frac{4.237g}{2.5cm^{3}}=1.6948$$

As rounding off the number, we get density = 1.7

4.(D) Rounding off 2.745 to 3 significant figures it would be 2.74. Rounding off 2.735 to 3 significant figures it would be 2.74.

5. (C) Since for 50.14 *cm,* significant number = 4 and for 0.00025, significant numbers = 2.

6.(C) In equilibrium, if θ is the require angle,



Cylinder *A*:

$$mg\sin 60° = kx\cos(60° - \theta)$$

Cylinder *B*:

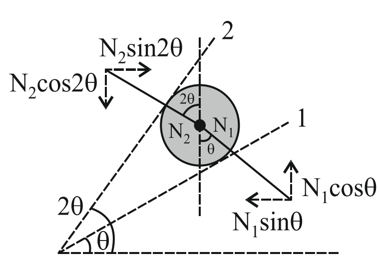
$$mg\sin 30° = kx\cos(30° + \theta)$$

$$= kx\sin(60° - \theta) \text{ on solving } \theta = 30°.$$

7.(C) In horizontal direction: $N_1\sin\theta = N_2\sin 2\theta$

$$N_1 = N_2 2\cos\theta$$

$$\frac{N_1}{N_2} = 2\cos\theta$$



8.(D) For upper cylinder

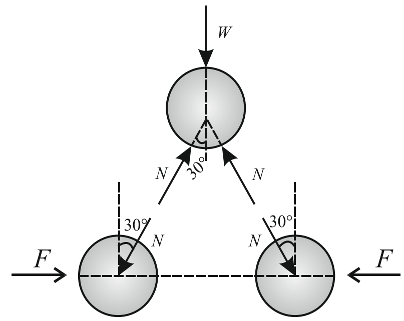
$$2N\cos 30° = W \qquad \ldots(i)$$

For lower block

$$N\cos 60° = F \qquad \ldots(ii)$$

From (i) & (ii)

$$\frac{2\cos 30°}{\cos 60°} = \frac{W}{F} \Rightarrow F = \frac{W}{2\sqrt{3}}$$



9. (ABCD)

$$I = a\left(1 - e^{-\frac{t}{\lambda}}\right)$$

Here, unit of I = unit of a = unit of $a\ e^{-\frac{t}{\lambda}}$

10.(ACD)

In various systems a physical quantity can have different units.

11. (ABCD)

$v = [LT^{-1}],\ a = [LT^{-2}],\ F = [MLT^{-2}]$

$\Rightarrow \quad \frac{v}{a} = T \qquad \Rightarrow \quad$ Unit of time $= \frac{5ms^{-1}}{20ms^{-2}} = \frac{1}{4}s$

As $F = ma$ so $m = \frac{F}{a} \qquad \Rightarrow \quad$ Unit of mass $= \frac{10N}{20\,ms^{-2}} = \frac{1}{2}kg$

Length $L = vt \qquad \Rightarrow \quad$ Unit of length $= (5ms^{-1})\left(\frac{1}{2}s\right) = \frac{5}{4}m$

Pressure $= \frac{F}{L^2} \Rightarrow$ Unit of pressure $= \frac{10N}{\frac{5}{4}\times\frac{5}{4}m^2} = \frac{32}{5}Pa$

12.(ABCD)

For any physical quantity, numerical value × unit – constant

For (a) $n_1u_1 = n_2u_2 \Rightarrow n_2 = \left(\frac{u_1}{u_2}\right)n_1 = \left(\frac{L_1}{L_2}\right)$

$$(500) = \left(\frac{1m}{1000\,m}\right)(500) = 0.5$$

For (b)

$$n_1 = \left(\frac{u_1}{u_2}\right)n_1 = \left(\frac{T_1}{T_2}\right)(n_1) = \left(\frac{1s}{3600s}\right)(7200) = 2$$

For (c)

$$n_1 = \left(\frac{M_1L_1^2T_1^{-2}}{M_2L_2^2T_2^{-2}}\right)(n_1)$$

$$=\left[\frac{(1kg)(1m)^2(1s)^2}{(1000kg)(1000m)^2(3600s)^{-2}}\right]\left(\frac{1}{36}=3.6\times10^{-14}\right)$$

For (d)

$$n_1=\left(\frac{M_1L_1T_1^{-2}}{M_2L_2T_2^{-2}}\right)(n_1)$$

$$=\left[\frac{(1kg)(1m)(1s)^2}{(1000kg)(1000m)(3600s)^{-2}}\right]\left(\frac{1}{36}\right)=0.36$$

13.(ACD)

Argument of logarithmic function must be dimensionless.

14.(AB)

(A) $\frac{bt}{2m}=[M^0L^0T^0]\Rightarrow[t]=\left[\frac{m}{b}\right]$

(B) $[b]=[a]=[L]\Rightarrow[\omega t]=\left[M^0L^0T^0\right]$

$\therefore \quad [\omega]=[T^{-1}] \qquad \therefore \quad [a_0\omega]=[LT^{-1}]$

15.(ABCD)

$$y=\frac{a}{t^2}e^{-\left[\frac{t}{b}-\frac{x}{c}\right]}$$

$$\left[\frac{t}{b}\right]=\left[\frac{x}{c}\right]=M^0L^0T^0$$

$\Rightarrow \quad [b]=T$, $[C]=[L]$ and $\left[\frac{a}{t^2}\right]=y$

$\Rightarrow \quad [a]=[LT^{-2}]$ and $e^{-\left[\frac{t}{b}-\frac{x}{L}\right]}$ is dimensionless

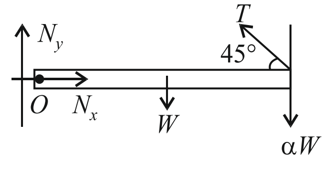
16.(ABCD)

Here $[XY]=L$ and $\left[\frac{Z}{XW}\right]=L\Rightarrow\left[\frac{X^2YW}{Z}\right]$ is dimensionless.

17.(ABD)

F>B.D. of the beam,

Considering rotational equilibrium about O.



$$T\sin45°\times l=\left(W\times\frac{l}{2}\right)+(\alpha W\times l)$$

$$T\times\frac{l}{\sqrt{2}}=\left(\frac{W}{2}+\alpha W\right)l$$

$$T=W\left(\frac{1}{\sqrt{2}}+\sqrt{2}\alpha\right)$$

(D) To break the string $T > 2\sqrt{2}W$ $\Rightarrow$ $W\left(\frac{1}{\sqrt{2}}+\sqrt{2}\alpha\right) > 2\sqrt{2}W \Rightarrow \alpha > \frac{3}{2}$

(C) $\alpha = 0.5 = \frac{1}{2}, T = W\left(\frac{1}{\sqrt{2}}+\sqrt{2}\times\frac{1}{2}\right) = \sqrt{2}W$

(B) $N_x = T\cos 45° = \sqrt{2}W \times \frac{1}{\sqrt{2}}$

(A) $N_v + T\sin 45° = W + \alpha.W$

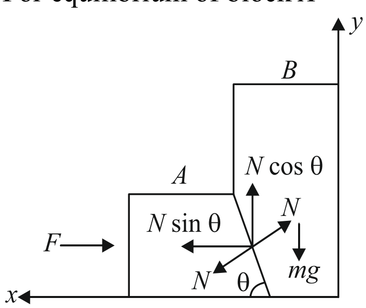
$N_v + \frac{W}{2} + \alpha W = W + \alpha W$; $N_v = \frac{W}{2}$ It does not depend on α.

18.(A)

19.(B) $\rho = ML^{-3}, G = M^{-1}L^3T^{-2}$ $\therefore$ $[\rho] = ML^{-1}T^{-2} = \rho^2L^2G$

$[V] = LT^{-1} = L\sqrt{\rho G}$

20.(C) For equilibrium of block A



$F = N\sin\theta$

$N = F/\sin\theta$

To lift block B from ground

$N\cos\theta \geq mg$ $\Rightarrow$ $\frac{F}{\sin\theta}\cos\theta \geq mg$

$F \geq mg\tan\theta = mg\left(\frac{3}{4}\right)$; So, $F_{\min} = \frac{3}{4}mg$

21.(C) If both the blocks are stationary.

Balancing forces along x-direction

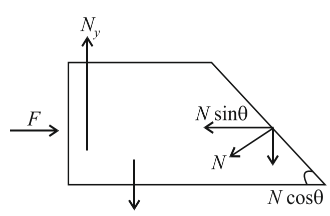
$F = N\sin\theta$

$\Rightarrow N = F/\sin\theta$

Balancing forces along y-direction

$N_y = mg + N\cos\theta$

$= mg + \left(\frac{F}{\sin\theta}\right)\cos\theta = mg + F\cot\theta$



$N_y = mg + \frac{4F}{3}$

22.(A)$\rightarrow$(S); (B)$\rightarrow$(P); (C)$\rightarrow$(Q); (D)$\rightarrow$(T)

Velocity $[v] = [LT^{-1}]$, acceleration $[a] = [LT^{-2}]$

Pressure $[p] = [ML^{-1}T^{-2}]$

$\Rightarrow \quad [M]=[Pv^4a^{-3}],[L]=[v^2a^{-1}]$ and $[T]=[va^{-1}]$

$\Rightarrow \quad 1 \text{ rilogram} = \dfrac{(10)^3\left(3\times10^8\right)^4}{(10)^3} \text{kilogram} = 81\times10^{34} \text{ kilogram}$

$1 \text{ reter} = \dfrac{\left(3\times10^8\right)^2}{10} \text{meter} = 9\times10^{15}$ meter and

$1 \text{ recond} = \dfrac{3\times10^8}{10} \text{second} = 3\times10^7$ second

$\Rightarrow \quad 1 \text{ kilogram} = \dfrac{1}{81}\times10^{-34}$ rilogram,

$1 \text{ meter} = \dfrac{1}{9}\times10^{-15}$ reter and 1 second record

For (A) : 1800 meter = $1800\times\dfrac{1}{9}\times10^{-15} = 2\times10^{-13}$ reter

For (B) : 3000 second = $3000\times\dfrac{1}{3}\times10^{-7} = 10^{-4}$ recond

For (C) : 8100 kilogram = $8100\times\dfrac{1}{81}\times10^{-34} = 10^{-32}$ rilogram

For (D) : 7200 joule: 7290 $kg\,m^2s^{-2}$ = $(7290)\left(\dfrac{1}{81}\times10^{-34}\right)\left(\dfrac{1}{9}\times10^{-15}\right)^2\left(\dfrac{1}{3}\times10^{-7}\right) = 10^{-50}$ roule

23.(A) (i) $\dfrac{1}{3} = 0.333 = 0.3$

Thus, (i) $\rightarrow$ (q)

(ii) $\dfrac{1.00}{3.0} = 0.333 = 0.33$

Thus, (ii) $\rightarrow$ (r)

(iii) $2-0.3 = 2-0 = 2$

Thus, (iii) $\rightarrow$ (s)

(iv) $2 - 0.6 = 2 - 1 = 1$

Thus, (iv) $\rightarrow$ (p)

24.(2.0) $[T]=[G]^a[m_s]^b[r]^c$

$M^0L^0T = [M^{-1}L^3T^{-2}][M]^b[L]^c = M^{b-a}L^{3a+c}T^{-2a}$

$\Rightarrow \quad a=-\dfrac{1}{2}, b=-\dfrac{1}{2}, c=\dfrac{3}{2} \qquad \Rightarrow \quad a+b+2c=2$

25.(3.0) Let in new system, dimensions be M', L' and T'

$\therefore$ Given $100M'L'T'^{-2} = 100MLT^{-2}$ …(i)

$20M'L'T'^{-2} = 20\times100ML^2T^{-2}$ …(ii)

Solving (i) and (ii); $L' = 100L \qquad \therefore \qquad M'T'^{-2} = \dfrac{1}{100}MT'^{-2}$

$\therefore$ 10 surface tension (MT^{-2}) in SI unit = $10^3 M'T'^{-2} \qquad \therefore \qquad n = 3$

26.(3) Let $d = k\rho^a S^b f^c$

where k is dimensionless. Then,

$$[L]=\left[\frac{M}{L^3}\right]^a\left[\frac{mL^2T^{-2}}{L^2T}\right]^b\left[\frac{1}{T}\right]^c$$

$$[L]=\left[M^{a+b}L^{-3a}T^{-3b-c}\right]$$

Equating the powers of M and L, we get

$$b=\frac{1}{3}=\frac{1}{n}\Rightarrow n=3$$

27.(2.0) $LHS=\left[ML^{-2}T^{-2}\right]^n\left[M^nL^{2n}T^{-2n}\right]$

$$RHS=\left[p^2c^2\right]=\left[m_0^2c^4\right]=\left[M^2(LT^{-1})^4\right]=\left[M^2L^4T^{-4}\right]$$

According to homogeneity principle,

LHS = RHS

$\therefore \quad [M^nL^{2n}T^{-2n}]=[M^2L^4T^{-4}] \qquad \therefore \quad n=2$

28.(4.0)

Let new units of length, mass and time be L, M and T then 7.6 L = 76 m

$\Rightarrow \quad L=10\ m$

$10\ M=100\ kg$

$\Rightarrow \quad M=10\ kg$

$360\ T=10\ s$

$\Rightarrow \quad T=10\ s$

4 N = nF, where F is unit of force in new system.

$$n=\frac{4\,kg\,m/s^2}{10\,kg\,10\,m(10s)^{-2}}=4$$

29.(1.0) $v\propto p^xd^y \Rightarrow [v]=[p^xd^y] \Rightarrow [LT^{-1}]=[(ML^{-1}T^{-2})(ML^{-3})^y]$

By comparing powers of M, L and T

$x+y=0,-x-3y=1,-2x=-1 \qquad \Rightarrow \qquad x=\frac{1}{2},y=-\frac{1}{2}$

30.(2.0) $\omega=G^xm^yr^z$

$$\left(\frac{1}{T}\right)=(M^{-1}L^3T^{-2})^{x(M)^y(l)^2}$$

$$M^0L^0T^{-1}=M^{3x+y}L^{-2x}$$

$$-x+y=0$$

$$3x+z=0$$

$$-2x=-1$$

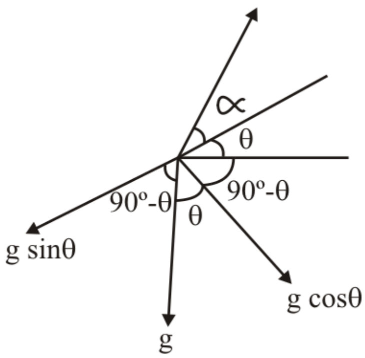
$$x=\frac{1}{2},y=\frac{1}{2},z=\frac{-3}{2}$$

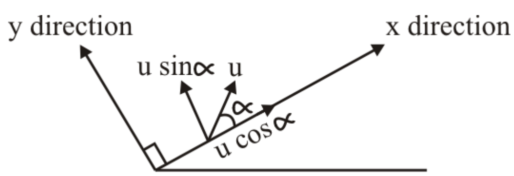
$$=\frac{1}{\frac{1}{2}+\frac{1}{2}+\left(-\frac{3}{2}\right)}=2$$

KINEMATICS AND 2 DIMENSIONS MOTION

1.(A) The particle is moving with constant acceleration therefore velocity time graph of the particle will be straight line. From t = 0 s to t = 1 s slope of given displacement-time graph is negative and decreasing. From t = 1 s to t = 2 s slope is positive and is decreasing. At time t = 0 and t = 2 s slope of displacement time graph is zero therefore velocity at that moment will also be zero.

2.(C)





$$T = \frac{2u_\perp}{a_\perp}$$

$$T = \frac{2\times(u\sin\alpha)}{g\cos\theta}$$

In both the cases , $u_\perp$ and $a_\perp$ is same, so the time $T_1 = T_2 = \frac{2u\sin\alpha}{g\cos\theta} = T$

(ii) $v = u + at$

$$u_x = u\cos\alpha + (-g\sin\theta)t$$

$$u_y = u\sin\alpha + (-g\cos\theta)t$$

For maximum height $\perp$ to the incline $U_y = 0$ $\quad\therefore\quad$ $u\sin\alpha = g\cos\theta t$

$$t = \frac{u\sin\alpha}{g\cos\theta} \quad ; \quad h = ut - \frac{1}{2}at^2$$

$$h = u\sin\alpha \times \left(\frac{u\sin\alpha}{g\cos\theta}\right) - \frac{1}{2}\times(g\cos\theta)\left(\frac{u\sin\alpha}{g\cos\theta}\right)^2$$

$$h = \frac{u^2\sin^2\alpha}{2g\cos\theta}$$

As u_y is same for both the cases and a_y is same so $h_1 = h_2 = \frac{u^2\sin^2\alpha}{2g\cos\theta}$

(iii) $R = (u\cos\alpha)T + \frac{1}{2}(-g\sin\theta)T^2$; As T is same for both

$u_2 = u\cos\alpha + g\sin\theta t$; $u_1 = u\cos\alpha - g\sin\theta t$

$$u_2 = u\cos\alpha + g\sin\theta \times \frac{u\sin\alpha}{g\cos\theta}$$

; Clearly $u_1 \neq u_2$

$$u_1 = u\cos\alpha - g\sin\theta \times \frac{u\sin\alpha}{g\cos\theta}$$

3.(B)

4.(A) The relative velocity V makes an angle θ with AB, where $\cos\theta = \frac{u}{V}$

The distance travelled during the period A arrives at nearest distance = $d\cos\theta$

$\therefore$ Required time = $\frac{d\cos\theta}{V} = \frac{du}{V^2}$ $\quad\therefore$ (A)

5.(B) P_1: A(5, 3) $\quad P_2$: (B) (7, 3)

$v_1 = 2\hat{i} + 3\hat{j} \qquad v_2 = x\hat{i} + y\hat{j}$

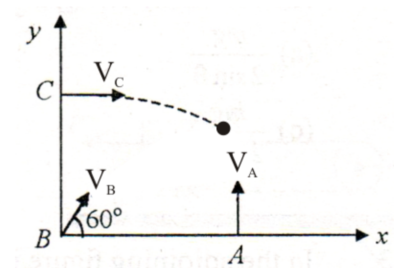
$s_1 = 5\hat{i} + 3\hat{j} + (2\hat{i} + 3\hat{j})t \quad | \quad s_2 = 7\hat{i} + 3\hat{j} + (x\hat{i} + y\hat{j})t$

$\Longrightarrow s_2 = (5 + 2t)\hat{i} + (3 + 3t)\hat{j} \quad | \Longrightarrow s_2 = (7 + xt)\hat{i} + (3 + yt)\hat{j}$

At t = 2 seconds they collide. It means that their S is same.

$\therefore \quad 5 + (2\times 2) = 7 + x\times 2, \quad 3 + (y\times 2) = 3 + (3\times 2) \quad \Rightarrow \quad x = 1, y = 3$

6.(B) Displacements of B and C in horizontal direction is same.



$\therefore \quad V_C = V_B\,(\cos 60^o)$

$\frac{v_C}{v_B} = \frac{1}{2}$ **......(1)**

Displacement of A and B in vertical direction is same to

$v_A \times t = v_B \times \sin 60^o \times t$

$\frac{v_A}{v_B} = \frac{\sqrt{3}}{2}$ **....(2)**

From (2) and (1)

$\therefore \quad v_A : v_B : v_C = \sqrt{3} : 2 : 1$

7.(B) $a = v\frac{dv}{dx} \Rightarrow \int v\,dv = \int a\,dx \qquad \Rightarrow \left[\frac{v^2}{2}\right]_{v_i}^{v_f}$ = Area under a –x curve.

8.(C) **(1)** As the bird starts it's motion at the same time as the ball, therefore, $V_x = u$.

(2) For all values of 'x' except h = 'Hmax", the ball will touch the bird twice.

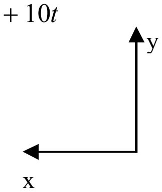
$\therefore \quad h = H_{max} = \frac{u_y^{\,2}}{2g} \qquad$ Range = $2\frac{(v_x)(v_y)}{g} = 2u\sqrt{\frac{2h}{g}}$

9.(C) $S_x = -\frac{3u}{5}t + \frac{a}{2}t^2 \qquad ; \qquad S_y = \frac{4u}{5}t - 5t^2$

$\Rightarrow \quad \frac{-3u}{5}\cancel{t} + \frac{a}{2}t^{\cancel{2}} = \frac{4u}{5}\cancel{t} - 5t^{\cancel{2}} \qquad \Rightarrow \qquad \frac{at}{2} + 5t = \frac{7u}{5} \qquad \Rightarrow \qquad at + 10t = \frac{14u}{5}$

$V_x = \frac{-3u}{5} + at \quad ; \qquad -V_y = \frac{4u}{5} - 10t \quad ; \qquad \frac{-3u}{5} + at = \frac{-4u}{5} + 10t$

$\left.\begin{aligned} 10t - at &= \frac{u}{5} \\ 10t + at &= \frac{14u}{5} \end{aligned}\right]$ add. $\quad ; \qquad 20t = 3u$



$\Rightarrow \quad 10\cancel{t} = a\cancel{t} = \frac{20\cancel{t}}{3 \times \cancel{5}} \Rightarrow a = 10 - \frac{4}{3} = \frac{26}{3}\ m/s \Rightarrow \quad a = \frac{26}{3} m/s^2$

10.(ABCD) Change in velocity = final velocity – initial velocity

$= u\cos\theta\hat{i} - \left(u\cos\theta\hat{i} + u\sin\theta\hat{j}\right) = -u\sin\theta\hat{j}$ $\therefore$ (A) is correct

Average velocity = (total displacement)/(time taken) = $\left(R\hat{i} / \text{Time of flight}\right)$

$= u\cos\theta\hat{i}$ $\therefore$ (B) is correct.

Change in velocity = final velocity – initial velocity

$= \left(u\cos\theta\hat{i} - u\sin\theta\hat{j}\right) - \left(u\cos\theta\hat{i} + u\sin\theta\hat{j}\right) = -2u\sin\theta\hat{j}$ $\therefore$ (C) is also correct.

Rate of change of momentum = force

Constant gravitational force is acting on the projectile. $\therefore$ (D) is also correct.

11.(AC) At $t = 2$ sec, projectile reverses its motion.

Maximum displacement in initial velocity = $\left(10 \times 2 + \left(\frac{1}{2}\right)(-5)(2)^2\right)$ = 10 m.

Distance travelled = Displacement from t = 0 to t = 2 added to magnitude of displacement from t = 2 to t = 3 = 12.5m

12.(BC) $a = V\frac{dv}{ds}$

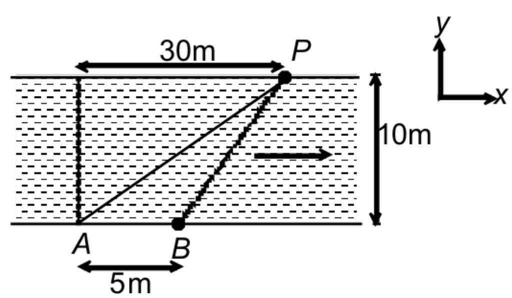
Or, $\int vdv = \int ads = \text{area under } a - \text{curve}$ Or, $\frac{v_f^2 - v_i^2}{2} = \left(\frac{1}{2} \times 10 \times 2 + 10 \times 4\right)[\text{For } S = 10m]$

$\Rightarrow \quad v_f = 10 m/s \quad [\because v_i = 0]$ $\therefore$ B is correct.

Max. velocity is attained at S = 30. $\therefore \frac{V_{max}^2 - V_i^2}{2} = \frac{1}{2} \times 30 \times 6 \quad \Rightarrow \quad v_{max} = 13.4$

13.(BD) For A : $V_{A,y} = \frac{\text{river width}}{\text{time}}$

$= \frac{10}{120} = \frac{1}{12} m/s$



And, $V_{A,x,\text{ river}} = 0 \quad \therefore V_{A,\text{ river}} = \frac{1}{12} m/s$

$V_{A,x,earth} = V_r + V_{A,x,river}$

= Vr

$\therefore 120 = \frac{30}{V_{A,x,earth}} = \frac{30}{V_r} \quad \Rightarrow V_r = \frac{1}{4} m/s$

$\therefore$ B is correct.

For B : $V_{B,y,earth} = \frac{10}{120} = \frac{1}{12} m/s$

$V_{B,x,earth} = \left(V_{B,x,river} + V_r\right) = \frac{25}{120} = \frac{5}{24} m/s$

14.(BD) For first one Minute :

$h_1 = 0 + \frac{1}{2} \times 10 \times (60)^2 = 18{,}000$m = 18 km

$V_1 = 0 + 10 \times 60 = 600\ m/s$

After first one minute : Rocket motion is under gravity.

$$V^2 = V_1^2 - 2gh_2 \quad \Rightarrow \quad h_2 = \frac{v_1^2}{2g} = \frac{(600)^2}{2\times 10} = 18000\text{m} \ (V = 0) = 18 \text{ km}$$

$\therefore$ Max. ht. reached , $H_{max} = h_1 + h_2 = 18+18 = 36$ km

Using $\vec{h} = \vec{u}\times t + \frac{1}{2}\vec{g}t^2 \quad \Rightarrow \quad 18000 = -600\times t_2 + \frac{1}{2}(+10)\times t_2^2$

$\Rightarrow \quad t_2^2 - 120t_2 - 3600 = 0 \quad \Rightarrow \quad t_2 = \left(60+60\sqrt{2}\right).$

$\therefore$ Total time required, $T = t_1 + t_2 = 60 + \left(60+60\sqrt{2}\right) = \left(120+60\sqrt{2}\right)$r

15.(ACD) $\vec{v}_{A,Board} = \vec{v}_{A,earth} - \vec{v}_{Board,earth} = 2v - v = v$

$\vec{v}_{B,Board} = -2v - v = -3v \quad \therefore \quad T = \frac{L}{\vec{v}_{A,B}} = \frac{L}{v-(-3v)} = \frac{L}{4v}$

$d_{B,Board} = V_{B,Board}\times T = 3v\times\frac{L}{4v} = \frac{3L}{4}$ and $d_{A,Board} = V_{A,Board}\times T = v\times\frac{L}{4v} = \frac{L}{4}$

16.(ABC) $\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \left(60\cos 60^\circ\,\hat{i} + 60\sin 60^\circ\,\hat{j}\right) - \left(60\hat{i}\right) = \left(-30\hat{i} + 30\sqrt{3}\hat{j}\right)$

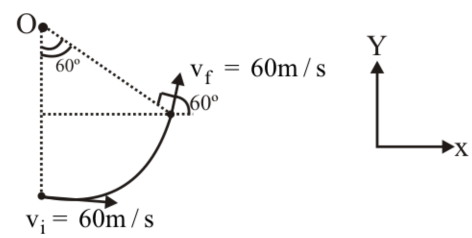
$\therefore \quad |\Delta\vec{v}| = \sqrt{(30)^2 + \left(30\sqrt{3}\right)^2} = 60$ m/s

$\vec{a} = \vec{a}_r + \vec{a}_t$

$a = \sqrt{a_r^2 + a_t^2} = a_r \quad \left[a_t = \frac{dv}{dt} = 0\right]$

$= \frac{v^2}{R} = \frac{(60)^2}{0.3\times 1000} = 12\text{m/s}^2$

$\left|\vec{a}_{arg}\right| = \frac{\left|\vec{v}_f - \vec{v}_i\right|}{\Delta t} = \frac{60}{\left(\frac{\pi}{3}\times 300\right)/60} = 11.5\text{m/s}^2$



17.(ABCD) $\vec{v}_A = 4\hat{i} + 4\hat{k}$

$\vec{v}_B = 3\hat{j} + 4\hat{k}$

$\therefore \vec{v}_{A,B} = \left(4\hat{i}+4\hat{k}\right) - \left(3\hat{j}+4\hat{k}\right) = 4\hat{i} - 3\hat{j} \quad \therefore \left|\vec{v}_{A,B}\right| = 5$ m/s (A is correct)

As the initial velocity and acceleration of both the particles in vertical direction are equal, so they would hit the ground at same time, B is correct.

Time of projectile = $\frac{2\times 4}{10} = \frac{4}{5}$sec Distance covered by A = $4\times\frac{4}{5} = \frac{16}{5}$m

Distance covered by B = $3\times\frac{4}{5}\text{m} = \frac{12}{5}$m. Separation = $\sqrt{\left(\frac{16}{5}\right)^2 + \left(\frac{12}{5}\right)^2} = \sqrt{\frac{16^2+12^2}{5}} = \frac{20}{5} = 4$m

C is correct.

$\vec{r}_{A,B} = \vec{r}_A - \vec{r}_B = \vec{v}_A\times t - \vec{v}_B\times t = \vec{v}_{A,B}\times t = \left(4\hat{i} - 3\hat{j}\right)\times t$

18.(ACD) $H_A = \frac{2}{3}R_B \Rightarrow \frac{u^2\sin^2\alpha}{2g} = \frac{2}{3}\frac{u^2\sin^2 p}{g} \Rightarrow 3\sin^2\alpha = u\sin\beta \Rightarrow 3\left(1-\cos^2\alpha\right) = 8\sin^2\beta$

$\sin^2\alpha = 1 \Rightarrow \beta_{max} = \frac{1}{2}\sin^{-1}\frac{3}{4}$; $R_{A\,max} = \frac{u^2\sin 90^\circ}{g} = \frac{u^2}{g}$

19.(ABD) If u is the initial speed and θ the angle of projection. Then $v_y = u\sin\theta - gt$ *i.e.*, $v_y - t$ graph is a straight line with negative slope and positive intercept. $x = (v\cos\theta)t$ *i.e.*, $x - t$ graph is a straight line passing through the origin. $y = (u\sin\theta)t - \frac{1}{2}gt^2$ *i.e.*, $y - t$ graph is parabola *i.e.*, $v_x - t$ graph is a straight line parallel to t-axis.

20.(ABC) $\frac{1}{2} = u_1 t + \frac{1}{2}a_1 t^2$ …(1)

and $-\frac{1}{2} = -u_1 t + \frac{1}{2}(-a_2)t^2$

$\Rightarrow \frac{1}{2} = u_2 t + \frac{1}{2}a_2 t^2$ …(2)

Subtracting (1) and (2), we get

$t = 2\left(\frac{u_2 - u_1}{a_1 - a_2}\right)$ …(3)

Substituting (3) in (1) or (2) and rearranging, we get

$1 = \frac{4(u_2 - u_1)}{(a_1 - a_2)^2}(a_1 u_2 - a_2 u_1)$ …(4)

Since the particle P and Q reach the other ends of A and B with equal velocities say v.

For particle P $v^2 - u_1^2 = 2a_2 1$ …(5)

For particle Q $v^2 - u_2^2 = 2a_1 1$ …(6)

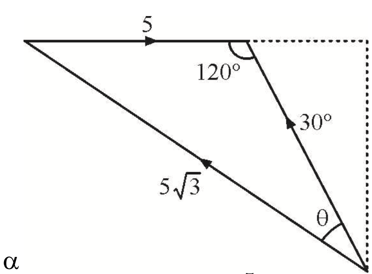
Subtracting and then substituting value of 1 and rearranging, we get $(u_2 + u_1)(a_1 - a_2) = 8(a_1 u_2 - a_2 u_1)$

21.(ABCD) $t = \frac{l}{u_{\text{rel}}} = \frac{l}{u + \frac{u}{2}} = \frac{2l}{3u}$

Distance $= \int v\,dt = u \times \frac{2l}{3u} = \frac{2l}{3}$

$v_{\text{avg}} = \frac{\text{total disp}}{\text{total time}} = \frac{l/\sqrt{3}}{2l/3u} = \frac{\sqrt{3}u}{2}$; $v_{AB} = u + u\cos 60 = \frac{3u}{2}$

22.(ABC) The velocity of motor boat is given as $\vec{v}_m = \vec{v}_{mw} + \vec{v}_w$



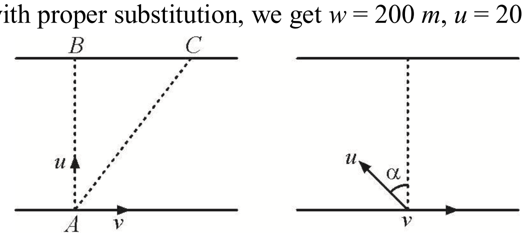
$\Rightarrow \frac{5}{\sin\theta} = \frac{5\sqrt{3}}{\sin 120°}$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30°$ $\therefore$ (A), (B) and (C)

23.(BC) In the first case $BC = vt_1$ and $w = ut_1$. In the second caste $u\sin\alpha$

Solving these four equations with proper substitution, we get $w = 200\ m$, $u = 20\ m/\text{min}$, $v = 12\ m/\text{min}$ and $\alpha \simeq 37°$



24.(AC) Distance travelled by motor bike at $t = 18\ s$

$$S_{\text{bike}} = S_1 = \frac{1}{2}(18)(60) = 540\ m$$

Distance travelled by car at $t = 18\ s$

$$S_{\text{car}} = S_2 = (18)(40) = 720\ m$$

Therefore, separation between them at $t = 18s$ is $180\ m$. Let separation between them decreases to zero at time t beyond $18s$. Hence, $S_{\text{bike}} = 540 + 60t$ and $S_{\text{car}} = 720 + 40t$

$$S_{\text{car}} - S_{\text{bike}} = 0 \quad \Rightarrow \quad 720 + 40t = 540 + 60t$$

$t = (18 + 9)\ s = 27s$ from start and distance travelled by both is $S_{\text{bike}} = S_{\text{car}} = 1080\ m$

25.(ACD) $v.\dfrac{dv}{dx} = -\alpha v \quad \Rightarrow \quad or \displaystyle\int_{v_0}^{0} dv = -\alpha \int_{0}^{x_0} dx$

$$v_0 = \alpha x_0 \Rightarrow x_0 = \frac{v_0}{\alpha}\ ;\ \frac{dv}{dt} = -\alpha v\ (or)\ \int_{v_0}^{v} \frac{dv}{v} = -\alpha \int_0^t dt$$

$$v = v_0 e^{-\alpha t}\ (or)\ v = 0 \text{ for } t = \infty \quad \Rightarrow \quad v = \frac{v_0}{e} \text{ when } t = \frac{1}{\alpha}$$

26.(ABC) Acceleration $= \dfrac{dv}{dt} = \dot{v} = 0 + kx$

$$\left\{\because\ \dot{x} = \frac{dx}{dt} = v\right\} \Rightarrow \dot{v} = a = kv = k(v_0 + kx)$$

Further, $a = \dfrac{dv}{dt} = kv \Rightarrow \dfrac{dv}{dt} = kv \Rightarrow \dfrac{dv}{v} = kdt \Rightarrow \displaystyle\int_{v_0}^{v_1} \frac{dv}{v} = k\int_0^t dt \Rightarrow t = \frac{1}{k}\log_e\left(\frac{v_1}{v_0}\right)$

Since, $v = v_0 + kx$. Hence slope of velocity displacement curve is $\dfrac{dv}{dx} = k$.

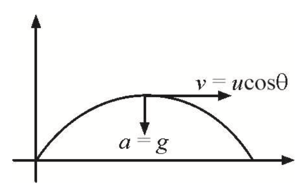
27.(AC) At highest point angle between $\vec{a}$ and $\vec{v}$ is zero. Hence, total acceleration is only normal or radial acceleration.

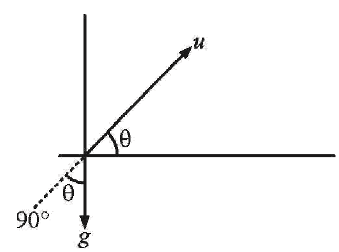
$\therefore \quad a = a_n = \dfrac{v^2}{R} \qquad \therefore \quad a = a_n = \dfrac{v^2}{R}$

But $\quad a = g \qquad$ but $\quad a = g$

$\therefore \quad g = \dfrac{(u\cos\theta)^2}{R} \qquad \therefore \quad g = \dfrac{(u\cos\theta)^2}{R}$

or $\quad R = \dfrac{u^2\cos^2\theta}{g} \qquad$ or $\quad R = \dfrac{u^2\cos^2\theta}{g}$





At point of projection component of acceleration (= g) along velocity vector is $-g\cos(90° - \theta)$ or $-g\sin\theta$.

28.(C) $V_{Rx} = 10\,m/s$

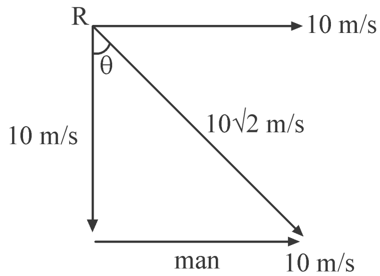
$V_{R_y} = 10\,m/s$

$V_{my} = 0$

Drops appear vertical to man $\quad \Rightarrow \quad V_{Rx} = V_{mx} \quad \Rightarrow \quad V_{mx} = 10\,m/s$

29.(D) $V'_{RY} = 10\sqrt{3}\,m/s$

$$V'_R = \sqrt{(V'_{Rx})^2 + (V'_{Ry})^2}$$



$= \sqrt{(10\sqrt{3})^2 + 10^2} = 20\ m/s$

30. **[A – q s ; B – r ; C – p ; D – q s]**

(A) constt. Speed

So, $\frac{dx}{dt} = \text{constant}$

$\therefore$ Position time graph will be straight line $\therefore$ B, D will be the match

(B) $\frac{dx^2}{dt^2} > 0$ $\therefore$ (C) is correct match

(C) $\frac{d^2x}{dt^2} = \text{'ve'}$ $\therefore$ (A) is correct match.

(D) $\frac{d^2x}{dt^2} = 0$ $\left(\frac{dx}{dt} = \text{constant}\right)$ $\therefore$ (B, D) are correct match.

31. **[A – q s ; B – p ; C – p ; D – qr]**

(A) $\frac{dv}{dx} \rightarrow \text{constant.}$

$\frac{vdv}{dx}$ is increasing uniformly $\Rightarrow$ acceleration is increasing (B) $a \propto x$ $\therefore$ (D)

(B) $\frac{dv^2}{dx} \rightarrow$ constant $\Rightarrow$ $2v\frac{dv}{dx} \rightarrow \text{constant}$

So; $\frac{vdv}{dx} \rightarrow \text{constant}$

Acceleration of particle is constant $\therefore$(A)

(C) $\frac{dv}{dt} \rightarrow \text{constant}$

$a \rightarrow$ constant α t $\therefore$ (A)

(D) $\frac{dv}{dt^2} \rightarrow$ constant or $\frac{dv}{dt}.\frac{dt}{dt^2} = \text{constant}$

$\frac{1}{2t}\frac{dv}{dt} = \text{constant}$

32. **[A – qs ; B – p ; C – r ; D – r]**

(A) $R = \frac{u^2 \sin 2\theta}{g} \Rightarrow \frac{R}{2} = \frac{(10)^2 \sin 60}{2g} = \frac{10\sqrt{3}}{4}$ $\Rightarrow$ $H_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{\left(10 \times \frac{1}{2}\right)^2}{2 \times 10} = \frac{10}{8}\text{m}$

Displacement $= \sqrt{\frac{R}{2}^2 + H_{max}{}^2}$ time $= \frac{u \sin\theta}{g} = \frac{1}{2}$sec. Avg. velocity $= \frac{\text{displacement}}{\text{total time}}$

(B) The time is given by $\frac{u\sin\theta}{g}$ Solve to get answer.

(C) $R=\frac{u^2\sin2\theta}{g}$ solve to get answer.

(D) Change in linear momentum = initial momentum – final momentum

$$=\sqrt{3}\left[10\cos30^o\hat{i}+10\sin30\hat{j}\right] - \sqrt{3}\times10\cos30\hat{i}$$

33. **[A – q ; B – r ; C – q ; D – r]**

If particle is gaining speed in a uniform manner, then it's tangential acceleration is non-zero and constant.

34.(20) At the time of collision, position of the both particles must be same.

So, diff. in x coordinate = 10 .

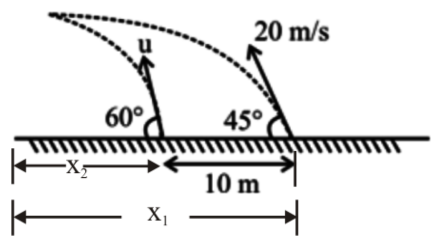
y coordinate is equal.

$x_1 - x_2 = 10.$

$20\cos45^o t - u\cos60^o t = 10$ …..(1)

and $u\sin60^o t = 20\cos45^o t$ …..(2)

Solving, we get the answer.



35.(1) Let t be the instant at which the ball hits rear face AB of the trolley.

Then $t=\frac{38}{v_0\cos45^o-u_0}=\frac{38}{28.28\cos45^o-10}=3.8s$

At t = 3.8 s, the y-coordinate of the ball is $y=(v_0\sin45^o)t-\frac{1}{2}gt^2=20t-5t^2$

Or $y = 20(3.8) - 5(3.8)^2 = 3.8$ m

Since 3.8 m > 2m, therefore, the ball cannot hit the rear face of the trolley.

Now, we assume that the ball hits the top face BC of the trolley, and let t' be that instant.

Then, $y = 2 = 20t' - 5t'^2$ or $t'^2 - 4t' + 0.4 = 0$; $t' = 3.9s$

Let d be the distance from the point B at which the ball hits the trolley. Then,

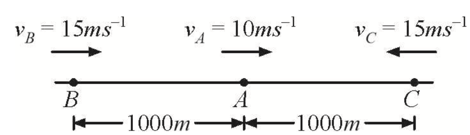
$d=(v_{0\cos}45^o-u_0)(t'-t)=(20-10)(3.9-9.8)=1m$

36.(5) $t_{min}=\left[\frac{2(\alpha+\beta)l}{\alpha\beta}\right]^{1/2}$ $v_{max}=\left(\frac{2\alpha\beta l}{\alpha+\beta}\right)^{1/2}$

$$t_{min}=\sqrt{\frac{(0.25+0.5)8\times10^3\times2}{0.25\times0.5}}=310s=5\min10s.$$

37.(1) The situation can be roughly shown in the figure. Let *C* take time *t* to overtake *A*.

$d_{rel}=1000\ m,\ v_{rel}=(10+15)=25\ ms^{-1}$



Here $t=\frac{d_{rel}}{v_{rel}}=\frac{1000}{25}=40s$

Let acceleration of *B* be a for overtaking

$d_{rel}=1000\ m;\ v_{rel}=15-10=5\ ms^{-1}$

$d_{rel}=a$ and $t = 40\ s$

Using $d_{rel}=u_{rel}t+\frac{1}{2}a_{rel}t^2$

$100=5\times40+\frac{1}{2}a(40)^2 \Rightarrow a=1\ ms^{-2}$

38.(8) $S_1 = \frac{1}{2}gt^2$; $S_2 = ut - \frac{1}{2}gt^2$

$S_1 + S_2 = h$; $4h = \frac{u^2}{2g} \Rightarrow u = \sqrt{8gh}$ $\Rightarrow$ $ut = h$

$\Rightarrow$ $\sqrt{8gh}\ t = h$ $\Rightarrow$ $t = \sqrt{\frac{h}{8g}}$

39.(4) $0 = v_0 \cos 30° - g \sin 30° t$

$\Rightarrow$ $t = \frac{v_0 \cos 30°}{g \sin 30°}$ ...(1)

$-H \cos 30° = -v_0 \sin 30° t - \frac{1}{2} g \cos 30° t^2$ ...(2)

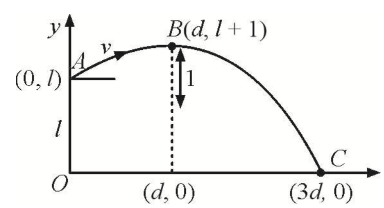
By equation (1) and (2), we get

$H = \frac{v_0^2}{g}\left[1 + \frac{\cot^2 \alpha}{2}\right] \Rightarrow v_0 = \sqrt{\frac{2gH}{5}} = 4\ m/s\ (\alpha = 30°)$

40.(3) V_{absolute} in vertically downward V_{He} after collision vertically upwards since collision is elastic so velocity of hail stones w.r.t. car before and after collision will make equal angles.

$\bar{V}_{He/1} = \bar{V}_H - V_c = \bar{V} - \bar{V}_1$; $\beta + 90 - 2\beta + \alpha_1 = 90$; $a_1 = \beta$. $2\beta = 2\alpha_1 \tan 2\beta = \tan 2\alpha_1 = \frac{V_1}{V}$

41.(3) The horizontal and vertical components of the velocity are the same, let it be $u = v\cos 45°$.



From A to B : $1 = \frac{u^2}{2g} \Rightarrow u^2 = 2g$

At B : $d = ut_1 \Rightarrow t_1 = \frac{d}{u}$; $1 = ut_1 - \frac{g}{2}t_1^2 = u\frac{d}{u} - \frac{g}{2}\frac{d^2}{u^2}$

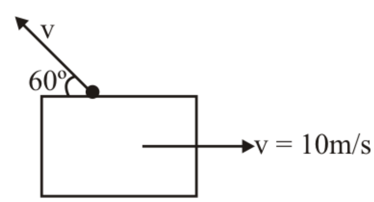
$\Rightarrow$ $1 = d - \frac{g}{2}\frac{d^2}{u^2}$ $\Rightarrow$ $1 = d - \frac{gd^2}{4g}$

$\Rightarrow$ $4 = 4d = d^2$ $\Rightarrow$ $d^2 - 4d + 4 = 0$ $\Rightarrow$ $d = 2m$; $3d = ut_2 \Rightarrow 1t_2 = \frac{3d}{u}$

$-l = ut_2 - \frac{1}{2}gt_2^2 = -u.\frac{3d}{4} - \frac{g}{2}\frac{9d^2}{4^2} = 3d - \frac{9gd^2}{4g} = 3d - \frac{9d^2}{4g} = 3 \times 2 - \frac{9}{4} \times 4 = 6 - 9 = -3$ $\Rightarrow$ $l = 3m$

42.(6) $T = \frac{2V_0 \sin(\theta - \alpha)}{g \cos \alpha} = \frac{2 \times 3\sqrt{3} \sin(60 - 30)}{10 \cos 30} = \frac{2 \times 3\sqrt{3} \times \frac{1}{2}}{10 \times \frac{\sqrt{3}}{2}} = 0.6 = 0.1x$; $x = 6$

43.(15) As seen (from ground, ball rises vertically, so;



$v\cos 60° = 10 m/s$ ......(1)

$v_y = v \sin 60°$

$H_{max} = \frac{u^2 \sin^2 \theta}{2u} = \frac{(v \sin 60)^2}{2g}$ ......(2)

Solving, we get the answer

44.(2) Range $= v_0 \times$ time of flight (t) $\Rightarrow$ $t = \frac{2v_0^2}{gv_0} = \frac{2v_0}{g}$

If y is the height of balloon at any instant t and $\frac{dy}{dt}$ its velocity then

$$y = -\left(\frac{dy}{dt}\right)\left(\frac{2v_0}{g}\right) + \frac{1}{2}g\left(\frac{2v_0}{g}\right)^2 \quad \Rightarrow \quad \frac{dy}{dt} + \frac{g}{2v_0}y - v_0 = 0 \quad ; \quad \frac{dy}{v_0 - \frac{g}{2v_0}y} = dt$$

$$\Rightarrow \quad -\frac{2v_0}{g}\ln\left(v_0 - \frac{g}{2v_0}y\right) = t + c \quad \text{At} \quad t = 0, y = 0 \quad \Rightarrow \quad c = -\frac{2v_0}{g}\ln v_0$$

Simplifying, we get $y = \frac{2v_0^2}{g}[1 - e^{-gt/2v_0}]$

45.(5) For rat $S = \frac{1}{2}\beta t^2$ ...(1)

For cat $S + d = ut + \frac{1}{2}\alpha t^2$ ...(2)

Putting the value of S from equation (i) in equation (2),

$(\alpha - \beta)t^2 + 2ut - 2d = 0$; $t = \frac{2u \pm \sqrt{4u^2 - 8d(\beta - \alpha)}}{2(\beta - \alpha)}$

For t to be real, $\frac{u^2}{2d} \geq (\beta - \alpha)$ $\therefore$ $\beta = \alpha + \frac{u^2}{2d}$

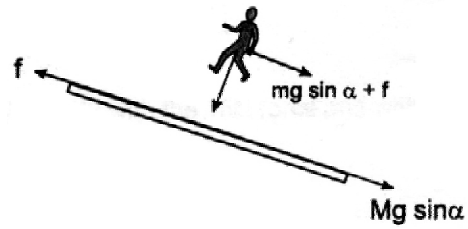
Substituting a, d and u we get

$\beta = 2.5 + \frac{5^2}{2 \times 5} = 2.5 + 2.5 = 5\ ms^{-2}$

DYNAMICS OF A PARTICLE

1.(C) For the equilibrium of block $\mu(150\cos 45^o + 50\cos 45^o) = 150\sin 45^o - 50\sin 45^o \Rightarrow \mu = 0.5$

2.(B) F.B.D. of man and plank are



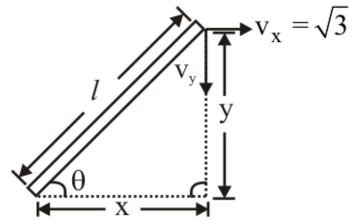
For plank be at rest, applying Newton's second law to plank along the incline $Mg\sin\alpha = f$ ……..(1)

And applying Newton's second law to man along the incline.

$Mg\sin\alpha + f = ma$ ………(2)

$a = g\sin\alpha\left(1 + \frac{M}{m}\right)$ down the incline

3.(B) $x^2 + y^2 = \ell^2$

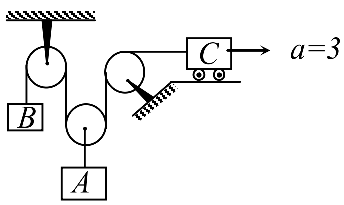


$\therefore \quad 2x\frac{dx}{dt} + 2y.\frac{dy}{dt} = 0 \quad \Rightarrow \quad \frac{dy}{dt} = \frac{-x}{y}, \frac{dx}{dt}$

$-v_y = -\cot 60^o \times \sqrt{3} \quad \Rightarrow v_y = 1 m/s$

$\therefore \quad v = \sqrt{{v_x}^2 + {v_y}^2} = 2 m/s$

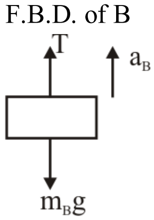
4.(C)



$m_A = 10kg.\ m_B = 5kg$

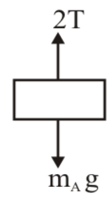
$\therefore$ let acc of A, B and C be a_A, a_B, a_C

F.B.D. of B



$T - 5g = 5a_B$ (1)

F.B.D. of A



$2T - 10g = 10a_A$ (2)

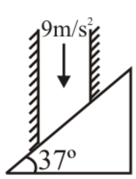
From (1) and (2)

$10a_A = 10a_B$

$a_A = a_B$

$a_A : a_B = 1:1$

5.(B)

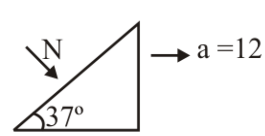


Final motion

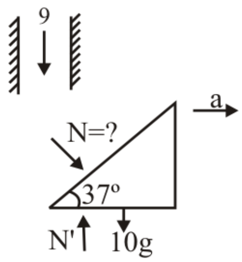
Velocity of block and wedge along contact will be same.

$\therefore \quad 9\cos 37^o = a\sin 37^o$

$a = 9 \times \frac{4}{3} = 12$



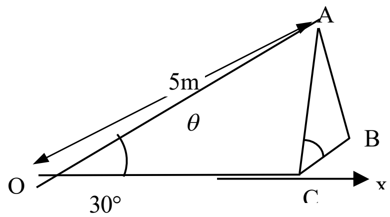
$N\sin 37^o = ma$



$N \times \frac{3}{5} = 12 \times 10 \quad \Rightarrow \quad N = 200N$

6.(B) Draw force diagram of M and se that net force on M in both the cases is zero.

7.(B)



Let B : foot of perpendicular drawn from A on the ground.

C : foot of perpendicular drawn from B to OX

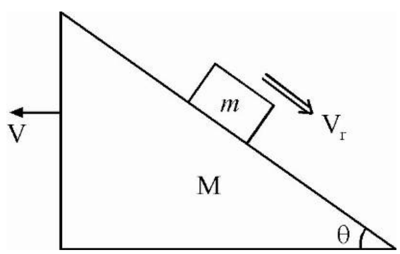
$AC = 5 \sin 30° = 2.5m$

$AB = AC \sin 30° = 1.25m$

$\Rightarrow \quad \sin\theta = \frac{AB}{OA} = \frac{1}{4} \quad \Rightarrow \quad$ acc. of the block $= g \sin\theta = 2.5 m/s^2$

$\left(s = ut + 1/2\, at^2\right) \quad \Rightarrow \quad \frac{1}{2}(2.5)t^2 \quad \Rightarrow \quad t = 2\,sec$

8.(B)

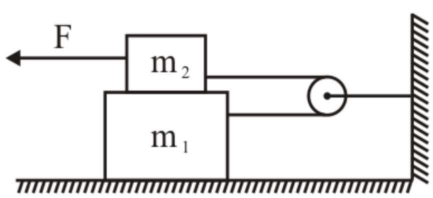


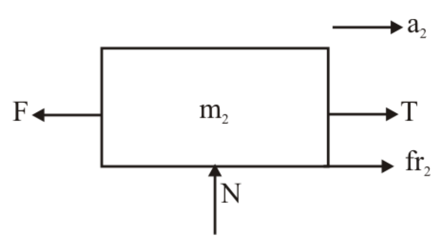
Along horizontal $MV = m\left(V_r \cos\theta - V\right)$

$\Rightarrow \quad V_r = \frac{10}{\sqrt{3}}\, m/s = 5.77\ m/s$

9.(BD)

10.(ACD)





Maximum fr force between m_2 and m_1

$= m_2 g\mu = 10 \times 0.2 \Rightarrow 2N$

$F - fr - T = m_2\, a_2$

If in equilibrium

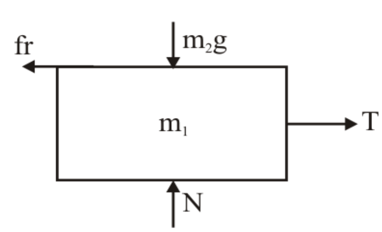
$F = fr + T$

$T < 2N \qquad [\text{If } F < 4N]$

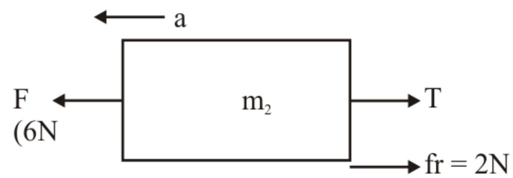
For m_1 in this case :

$T < Fr_{max}$

$\therefore$ fr and T will be equal $\quad \therefore \quad$ (A) is correct



It is not necessary if F > 4
T = 4N. (B) is not correct.
If F > 4, max Fr = 2N
∴ and system will accelerate
∴ system will not be in equilibrium (C) is correct

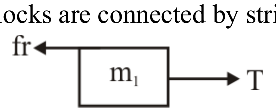


If F = 6N : Fr will be at max value. Fr = 2N

$F - T - Fr = a_2$

$4 - T = a_2$

Since blocks are connected by string there acceleration will be equal

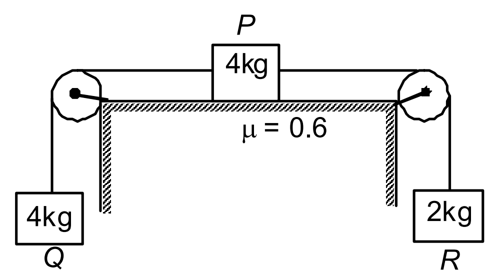


∴

$T - fr = a$ $\quad$ $T - 2 = a$ $\quad$ $4 - T = a$

$2a = 2,\ a = 1$ and, $T = 3N$

11.(ABCD)



If Fr is not present system will move towards right

∴ fr will act on P towards left

Max fr = $40 \times 0.6 = 24N$

System will be in equilibrium

If $m_q g = f_r + m_R g$

$40 = f + 20$

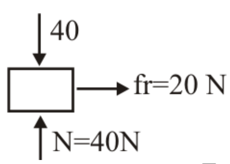
$f = 20N$

Since for is within limiting value ∴ $f = 20$ N system is in equilibrium

Q is in equilibrium R is in equilibrium

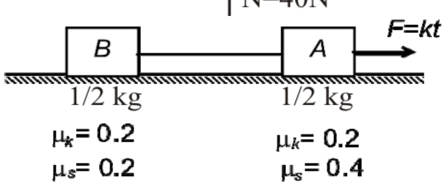
∴ $T_A = 20N$

∴ TB = 40 N



Contact force $= \sqrt{40^2 + 20^2} = \sqrt{2000} = 20\sqrt{5}$

12.(ABC)



System will be in equilibrium until A is in equilibrium.

Max Fr force on A $= \mu_s \times 0.5g$

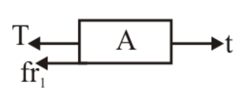
$= 2N$

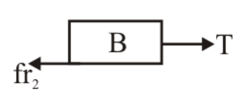
$F = Kt = t$ $\quad$ $(k = 1)$

If $T = f_2$ B is in equilibrium

f_2 max $0.2 \times 0.5g = 1N$

∴For $T \le 3$: system as a whole is in equilibrium





$\therefore$ For t upto 3 sec. system is in equilibrium and is at rest.

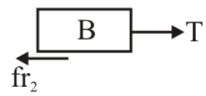
$\therefore$ Options A, B and C are correct.

For $t > 3$ sec :

F on A = 1N

Fr force max while motion of $A \Rightarrow \mu_k \times N = 1$

$$T - 1 = t - T - 1 = \frac{a}{2}$$



$$T - 1 = \frac{a}{2}$$

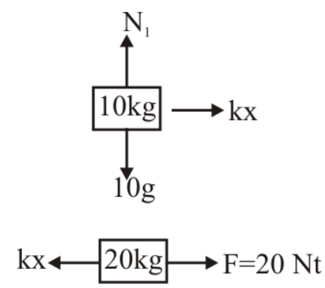
$$t - 2 = a = \frac{dv}{dt} \quad \text{or} \quad \int_3^{10} (t - 2)dt = \int_0^v dv$$

$$\left[\frac{t^2}{2} - 2t\right]_3^{10} = (V - 0) = 31.5 \text{ m/s}$$

$\therefore$ D is incorrect.

13.(BCD) To maintain constant velocity, $F_{net} = 0 \Rightarrow P = fr$ always

14.(BC) For 10kg block : $kx = 10 \times 12 = 120 Nt.$



For 20 kg block : $200 - kx = 20 \times a$

$$\Rightarrow \quad a = \frac{200 - 120}{20}$$

$= 4$ m/s^2.

15.(AD) Suppose blocks A and B move together. Applying NLM on C, A + B, and D

60 – T = 6a

T – 18 – T' = 9a

T – 10 = 1a

Solving a = 2 m/s^2

To check slipping between A and B, we have to find friction force in this case. If it is less than limiting static friction, then there will be no slipping between A and B.

Applying NLM on A.

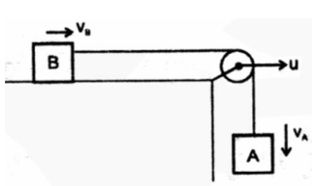
T – f = 6(2)

As T = 48N

f = 30 N

and f_s = 42 N hence A and B move together. And T` = 12 N.

16.(BD)



by string constrain

$v_A + u - v_B = 0$ Or $v_B + u - v_A$

Differentiating both side ; $a_B = 0 + a_A$

17.(AD) For equilibrium

$$\lambda Rg\int_0^{\pi/2}\cos\theta d\theta=\mu\lambda Rg\int_0^{\pi/2}\sin\theta d\theta$$

$\therefore \quad \mu=1$

At the position of maximum tension in the rope

$\lambda Rd\theta\cos\theta=\mu(\lambda Rd\theta g\sin\theta)$

$\therefore \quad \theta=45^\circ$

At any θ

$dT=\lambda Rd\theta\cos\theta-\mu\lambda Rd\theta g\sin\theta$

$$\int_0^{T_{max}}dT=\lambda Rg\int_0^{\pi/4}(\cos\theta-\sin\theta)d\theta$$

$$T_{max}=\lambda Rg[\sin\theta+\cos\theta]_0^{\pi/4}=\lambda Rg\left[\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right]=\lambda Rg(\sqrt{2}-1)$$

18.(AC) Initially the $4kg$ block experience increasing friction as it tries to prevent relative motion between the $2kg$ and $4kg$ and the force increases with time then there is a discontinuity in the graph of $a_4 vs t$ because the values of frictional force decrease from limiting to kinetic friction. The friction causes increasing acceleration on 2 kg block but after it starts relative motion the kinetic friction it constant causing constant acceleration.

19.(AD) Initially both friction and external forces acts opposite to motion.

$\mu mg=-5N$

$F=-15N \qquad V=at$

$a=-\frac{20}{10}m/s^2 \qquad$ At $t=\frac{10}{+2}=+5s$

$=-2\ m/s^2 \qquad$ Velocity change direction

Later after velocity changes direction friction acts opposite to motion (+ve x-axis) and ext force act along motion.

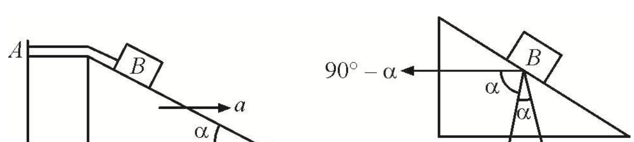
$\mu mg=5N$

$F=-15\ N$

$a=-\frac{10}{10}m/s^2=-1\ m/s^2$

Hence, $\frac{dv}{dt}=-2$ at $t=5$ and $\frac{dv}{dt}=-1$ from then $\frac{d^2x^2}{dt^2}=-2$

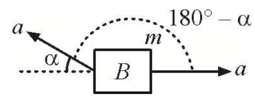
20.(BC)



When the block is seen with respect to wedge a pseudo force ma will act horizontal. By normal constrain

$ma\cos(90^\circ-\alpha)+mg\cos\alpha=N$

$ma\sin\alpha+mg\cos\alpha=N$



Magnitude of acceleration

$\sqrt{a^2 + a^2 + 2a^2 \cos(180° - \alpha)}$

$\sqrt{2a^2(1-\cos\alpha)}$

$\sqrt{4a^2 \sin^2 \alpha/2}$

$2a\sin\alpha/2$

21.(ABCD)

Taking wall as reference from a pseudo force acts on the block = ma

Where a is acceleration of reference $= 20\ m/s^2$

$N = ma$

$= 10 \times 20$

$= 200\ N$

Limiting friction $= \mu N$

$= 0.6 \times 200 = 120\ N$

Friction required to prevent sliding is 100 N

$f = 100\ N$

Total contact force $= \sqrt{(200)^2 + (100)^2}$

$= 100\sqrt{5}N$

22.(ACD)

For breaking off the plane : $\quad F\sin\alpha = mg$

$\Rightarrow \quad at_0^2 \sin\alpha = mg \quad \Rightarrow \quad t_0 = \sqrt{\dfrac{mg}{a\sin\alpha}}$

Speed at time of breaking off.

$$\Rightarrow \quad v = \int_0^{t_0} \frac{at^2\cos\alpha}{m} dt = \frac{at_0^2\cos\alpha}{3m} = \frac{a\cos\theta}{3m}\cdot\frac{mg}{a\sin\alpha}\sqrt{\frac{mg}{a\sin\alpha}} = \sqrt{\frac{mg^3}{9a\tan^2\alpha\sin\alpha}}$$

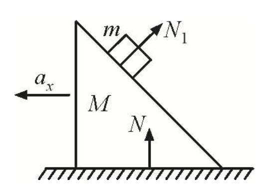
$$a = \frac{F\cos\alpha}{m} = \frac{at_0^2\cos\alpha}{m} = \frac{amg}{a\sin\alpha}\cdot\frac{\cos\alpha}{m} = g\cot\alpha.$$

$$s = \int v\,dt = \int_0^{t_0}\frac{at^3}{3m}\cos\alpha\,dt = \frac{a}{12m}t_0^4 = \frac{a}{12m}\cdot\frac{m^2g^2\cos\alpha}{a^2\sin^2\alpha} = \frac{mg^2}{12a\tan\alpha\sin\alpha}$$

23.(BD) As on the gravity and normal are the only two forces acting

$(M+m)g - N = (M+m)a_{cm}$

$g - \dfrac{N}{(M+m)} = a_{cm}$



By normal constrain

Block m will apply N_1 force of the wedge normally.

And the component of the force in x direction $(N_1 \sin\theta)$ will provide acceleration

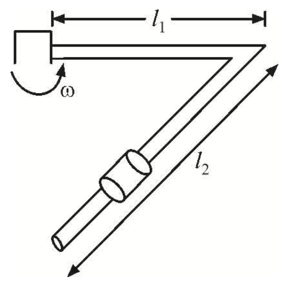
$Ma_x = N_1 \sin\theta$

$a_x = \dfrac{N_1 \sin\theta}{M}$

24.(BCD) Angular velocity of sleeve = ω

Radius of rotation = l_1

Centrifugal force $= m\omega^2 l_1$



N_x be normal in plank of motion $N_x = m\omega^2 l_1$

Let N_y be normal in direction planking out gravity

$N_y = mg$

$N^2 = N_x^2 + N_y^2$

$N = \sqrt{(m\omega^2 l_1)^2 + (mg)^2}$

$= \sqrt{m^2(\omega^4 l_1^2 + g^2)}$

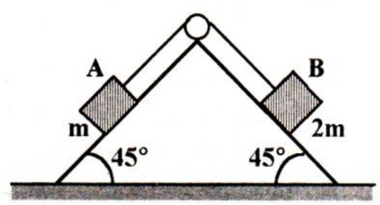
$= m\sqrt{\omega^4 l_1^2 + g^2}$

As it starts slipping

F = $f = \mu N$

25.(D) a = 0, since $\left[(m_B - m_A) g \sin 45^o < g(\mu_A m_A + \mu_B m_B)\cos 45^o\right]$

26.(B) Since mg sin 45º > $\dfrac{2}{3}\text{mg}\cos 45^o$



And 2mg sin 45º > $\dfrac{2}{3}\text{mg}\cos 45^o$

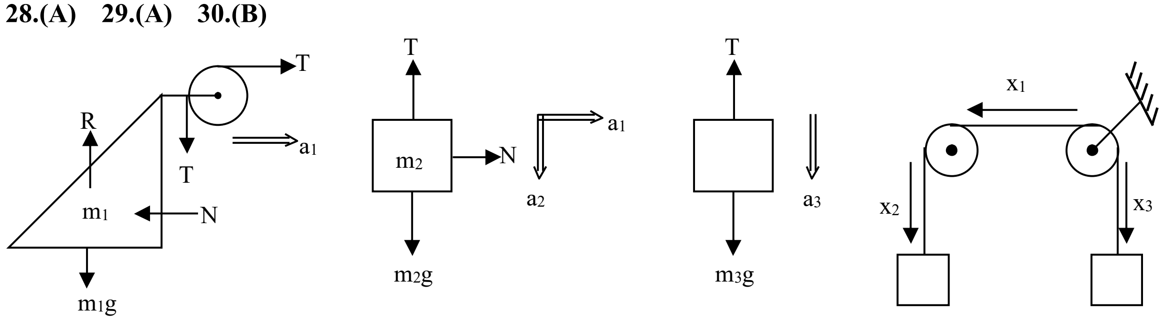
Therefore block B has tendency to move downward.

We have $\dfrac{2\text{mg}}{\sqrt{2}} - T - F_{r_B} = 0$

$F_{r_B} = \dfrac{2}{3}\dfrac{\text{mg}}{\sqrt{2}}$ $\quad\therefore\quad$ T = $\dfrac{4}{3}\dfrac{\text{mg}}{\sqrt{2}}$

27.(B) Again $T - \dfrac{\text{mg}}{\sqrt{2}} - F_{r_A} = 0$ $\quad\therefore\quad$ $F_{r_A} = \dfrac{\text{mg}}{3\sqrt{2}}$ downward.

28-30. **28.(A)** **29.(A)** **30.(B)**



$T - N = m_1 a_1$ . . . . (i) $\qquad N = m_2 a_1$ . . . . (ii)

$m_2 g - T = m_2 a_2$ . . . . **(iii)** $m_3 g - T = m_3 a_3$ . . . . **(iv)**

$x_1 + x_2 + x_3$ const.

$\Rightarrow \quad -a_1 + a_2 + a_3 = 0$ . . . . **(v)**

Solve the equation for T, a_1 and a_3 to get : $T = \frac{120}{7} N$; $a_1 = \frac{40}{7} m/s^2$; $a_3 = \frac{30}{7} m/s^2$

31. [A – p r] [B – p s] [C – p r] [D – q]

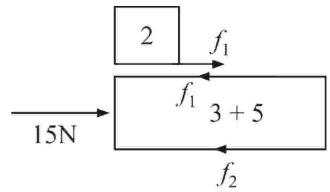
(A) $f_{1\ell} = 0.3 \times 20 = 6N,\ f_{1K} = 0.2 \times 20 = 4N$

$f_{2\ell} = f_{2K} = 0.1 \times 50 = 5N$

For combined block

$15 - 5 = 10a = a = 1 m/s^2$

$f_1 = 2 \times 1 = 2N$



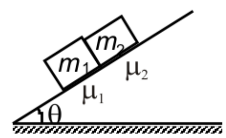
Hence all blocks will have same acceleration. Also $f_1 < f_\ell$ hence [A-p, r] similarly solve others

32. [A – q s] [B – p r] [C – q r] [D – q s]

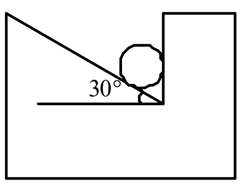
Acceleration will be same till N exists. This is possible only if $\mu_1 > \mu_2$

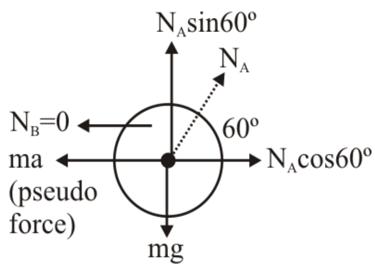
When acceleration will be different, then N = 0 and $\mu_2 > \mu_1$

Now match the option.



33.(10)



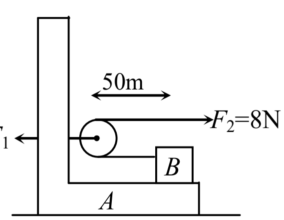


$N_A \sin 60° = mg$ …..(1) ….(2)

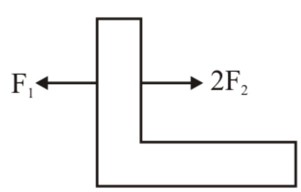
From (1) and (2) $N_B = 0$ incase of a_{max}.

$\therefore \quad \frac{mg}{\sqrt{3}} = ma \qquad a = \frac{g}{\sqrt{3}} \qquad \therefore \qquad n = 10$

34.(5)



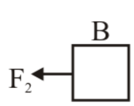
$\Rightarrow \angle$ON



For A :

$F_1 - 2F_2 = m_A a_A$

$A = 4 m/s^2$



For B :

$F_2 = m_B a_B \Rightarrow a_B = \frac{8}{1} = 8 m/s^2$

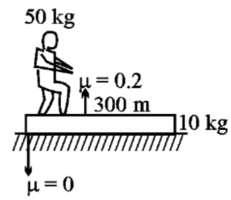
$\therefore$ Distance covered by B $\Rightarrow$ dist covered by A + 50 $\therefore$ $\frac{1}{2}\times 8\times t^2 = \frac{1}{2}\times 4\times t^2 + 50$

$2t^2 = 50$

$t^2 = 25$

$t = 5\text{sec.}$

35.(1) Max acceleration of system is possible if for value is max



Fr max = $500\times 0.2 = 100$ N

$\therefore$ max acceleration of man = 2m/s^2 and of plank = 10 m/s^2

Their acceleration will always be in opposite directions .

Now, let man acce at 2m/s^2 and plank at 10 m/s^2 for time t, and let them decelerate at 2 m/s^2 and 10 m/s^2 for time t_2 .

$\therefore$ $\frac{1}{2}2{t_1}^2 + \frac{1}{2}\times 10{t_1}^2 + \frac{1}{2}\times 2\times t^2 + \frac{1}{2}\times 10{t_2}^2 = 300$; $t_1^2 + t_2^2 = 50$

Now $t_1 + t_2$ will be min if time for acceleration and deceleration are same

$\therefore$ $t_1 = t_2$ $\therefore$ $2t_1^2 = 50$

$t_1^2 = 25\text{sec}$

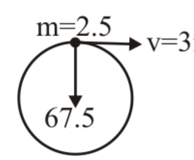
$t_1 = 5$ sec. $\therefore$ total time t $\Rightarrow$ $2t_1$ $\Rightarrow$ 10 sec.

36.(9) Particle in gravity free space

$m = 2.5\text{kg}$

F = 67.5N

This is a case of circular motion where F is $\perp$ to motion. Since it is uniform circular motion



$F = \frac{mv^2}{R}$

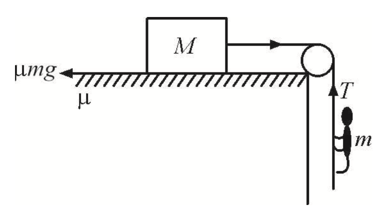
$67.5 = \frac{2.5\times 9}{R}$

$67.5 = m\frac{v^2}{R}$; $R = \frac{2.5\times 9}{67.5} = \frac{1}{3}$ $\therefore$ $\omega = \frac{v}{R} = 9\text{rad/s}$; $\therefore$ $T = \frac{2\pi}{w} = \frac{2\pi}{9}$

37.(4) $a_{\text{monkey/rope}} = g/4$

Let acceleration of $M = a_0$

So, acceleration of rope $= a_0$



$\vec{a}_{\text{monkey}} = \vec{a}_{\text{monkey/rope}} + \vec{a}_{\text{rope}} = (a_0 - g/4)$

Now for mass M, $T - \mu Mg = Ma_0$ ...(i)

For monkey $mg - T = m(a_0 - g/4)$ ...(ii)

From equation (i) and (ii)

$$mg - \mu Mg = a_0(M+m) - \frac{mg}{4}$$

$\Rightarrow$ $a_0 = \left(\frac{5m}{4} - \mu M\right)g/(M+m) = \frac{(5m - 4\mu M)g}{4(M+m)}$

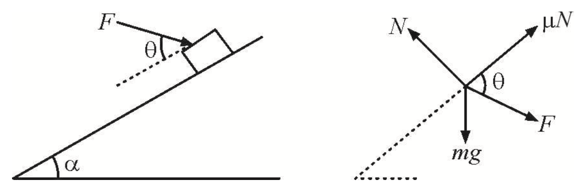
$\Rightarrow \quad T = \dfrac{M(5m - 4\mu M)g}{4(M+m)} + \mu Mg$

38.(1) In order to achieve a minimum of *F*, it should be directed as shown in the figure, The value of θ can be found using the *FBD* of the block. Normal to the incline.

$N = mg\cos\alpha + F\sin\theta \qquad \ldots\text{(i)}$

Along the incline

$\mu N + F\cos\theta = mg\sin\alpha \qquad \ldots\text{(ii)}$



From (i) and (ii), we get

$F = \dfrac{mg(\sin\alpha - \mu\cos\alpha)}{\cos\theta + \mu\sin\theta}$

For *F* to be minimum

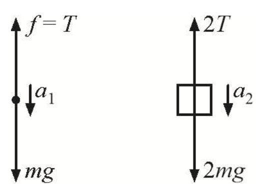
$\dfrac{d}{d\theta}(\cos\theta + \mu\sin\theta)$

$\Rightarrow \quad \mu = \tan\theta$

$\Rightarrow \quad F_{\min} = \dfrac{mg(\sin\alpha - \mu\cos\alpha)\sqrt{1+\mu^2}}{1+\mu}$

39.(6.25)

$mg - T = ma_1$



$2mg - 2T = 2ma_2$

So, $a_1 = a_2$

Relative acceleration of bead with respect to end = $3a$

$\therefore \quad$ displacement of block $x = \dfrac{1}{2}at^2 = \dfrac{l}{3} = 6.25m$

40.(2) At $V_{\max}, \dfrac{dv}{dt} = 0$ so there won't be any tangential force, only force will be radial provided by friction

$\dfrac{mv^2}{r} = \mu_0\left(1 - \dfrac{r}{R}\right)mg \quad ; \quad v^2 = \mu_0\left(r - \dfrac{r^2}{R}\right)g$

$\dfrac{d(v)^2}{dr} = 0 \quad ; \quad r = \dfrac{R}{2} = 1 \qquad \ldots\text{(i)}$

Therefore at $r = 1$, v is maximum

41.(2) Drawing *F.B.D.* diagrams

$10g - 2T = 10a_A$

$5g - T = 5a_B$

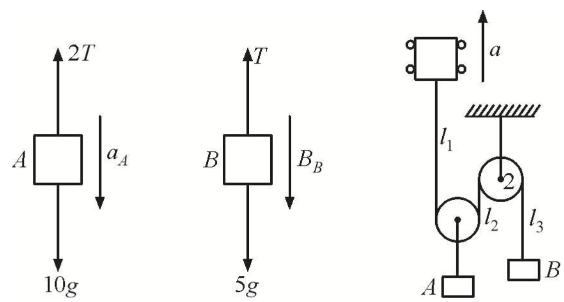
T from equations (1) & (2) we get

$10a_A - 10a_B = 0$

$\Rightarrow \quad a_A = a_B$

$l_1 + l_2 + l_3 =$ constant

$y_E + y_A + y_A + y_B =$ constant



Differentiation twice w.r.t. time,

$a + 2a_A + a_B = 0$

$2a_A + a_B = -3$ or $3a_0 = -3$

$\Rightarrow \quad a_B = -1 \text{ m/s}^2, a_A = -1 \text{ m/s}^2$

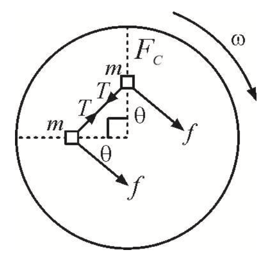
$a_A = a_B = 1 \text{ m/s}^2$ upwards

42.(2) For small values of ω friction will be directed radially inwards as the tension in the string is zero. The string will develop tension only if the centrifugal force F_C exceeds the limiting friction f_e i.e. when

$m\omega^2 r > \mu mg \qquad (f_e = \mu mg) \qquad$ or $\qquad \omega = \sqrt{\dfrac{\mu g}{r}}$

in this case direction of friction will be as shown in the figure.

For equilibrium



$F_0 = T\cos 45° + f_e \cos\theta$

and $T\sin 45° = f_e \sin\theta$

Eliminating T, we get

$F_c = f_e(\sin\theta + \cos\theta)$

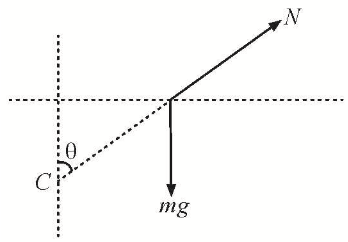
i.e., $m\omega^2 r = \mu mg\left[\sqrt{2}\sin(\theta + 45°)\right]$ or $\qquad \omega^2 = \dfrac{\sqrt{2}\mu g}{r}\sin(\theta + 45°)$

Maximum value of $\sin(\theta + 45°)$ is 1. $\qquad \therefore \qquad$ Maximum value of $\omega = \sqrt{\dfrac{\sqrt{2}\mu g}{r}} = 2.$

43.(2) Using conservation of energy principle, if v be the speed of either ball when its radius vector makes angle θ with vertically upward direction.

$mgR[1 - \cos\theta] = \dfrac{1}{2}mv^2 \qquad \Rightarrow \qquad \dfrac{mv^2}{R} = 2mg[1 - \cos\theta]$

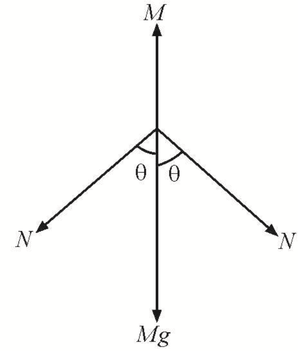
Free body diagram of ball



F.B.D. (i)

N is the normal reaction by tube walls on ball B.

Free body diagram of tube



F.B.D. (i)

N' be the normal reaction force by ground on tube.

From *F.B.D.* (i)

$$N = mg\cos\theta - \frac{mv^2}{R} = mg\cos\theta - 2mg[1-\cos\theta]$$

From *F.B.D.* (ii)

$$N' = 2N\cos\theta + Mg$$

At the instant tube breaks its contact with ground

$N' = 0 \qquad \Rightarrow \qquad Mg + [mg\cos\theta - 2mg(1-\cos\theta)]2\cos\theta = 0$

For $\theta = 60°$, we get $m/M = 2$.

44.(3) $mg\sin\theta - f = m\omega^2 l$

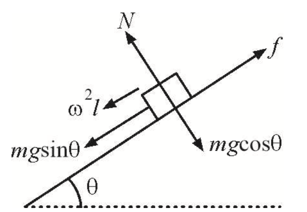
$mg\sin\theta - \mu mg\cos\theta = m\omega^2 l$

$g \times \frac{1}{2} - g\mu \times \frac{\sqrt{3}}{2} = 2 \times l$

$5 - 2 = 5\sqrt{3}\,\mu$

$\frac{\sqrt{3}}{5} = \mu$

$\mu^2 = \frac{3}{25} \qquad \Rightarrow \qquad k = 3$

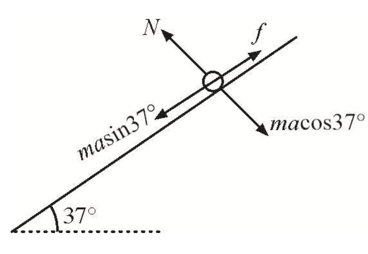


45.(3) Acceleration of bead $= (g\sin 37° - \mu g\cos 37°)$

$15.3 = ut + \frac{1}{2}at^2$

$15.3 = \frac{1}{2} \times (g\sin 37° - \mu g\cos 37°) \times t^2$

$t = 3$ sec



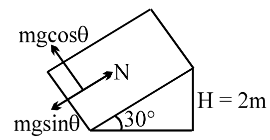
ENERGY AND MOMENTUM

1.(D) Since the external force is always equal and opposite to the tangential component of mg, there would be no acceleration of block. i.e. its kinetic energy will remain constant. Now, use work energy theorem. Since there is no change in K.E., net work done on the block would be zero. Therefore, work done by external force will be negative of the work done by gravity, i.e. –(mgH) = mgH.

2.(B) Use conservation of energy to find the speed of block at 60^0 position. Note that there is no change in spring energy. Now write the force equation at this position, putting normal reaction as zero.

3.(C) $N - mg\sin\theta = \dfrac{mv^2}{R}$

$\dfrac{1}{2}mv^2 = mgH \quad \Rightarrow \quad \dfrac{mv^2}{R} = \dfrac{2mgH}{R} = 2mg \quad \therefore \; N = \dfrac{mg}{2} + 2mg = \dfrac{5mg}{2}$



Contact force $= mg\sqrt{\dfrac{25}{4} + \left(\dfrac{\sqrt{3}}{2}\right)^2} = \dfrac{\sqrt{28}}{2}mg$

4.(B) In both COM and ground frame, Kmax is when x is zero in spring, which occurs simultaneously.

$\text{Vcm} = \dfrac{m(V)+0}{5m} = \dfrac{V_o}{5} \quad \Rightarrow \quad \text{Kmax cm} = \dfrac{1}{2}m\left(\dfrac{4V_o}{5}\right)^2 + \dfrac{1}{2}(4m)\left(\dfrac{V_o}{5}\right)^2 = \dfrac{2}{5}mv_o^2$

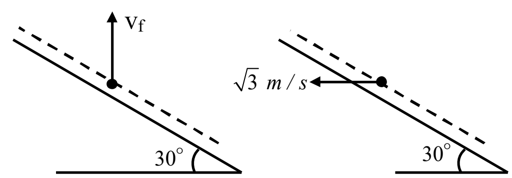
$\Rightarrow \quad \text{Kmax ground} = \dfrac{1}{2}mv_o^2 \quad ; \quad \text{Kmin cm} = 0, \text{ Kmin ground} = \dfrac{1}{2}(m+4m)V_{cm}^2 = \dfrac{V_o^2}{10}$

$\text{Kmax m} = \dfrac{1}{2}mV_o^2$ (ground frame)

Kmin m = 0 (ground frame when energy is shared by spring & 4m only) Hence (B)

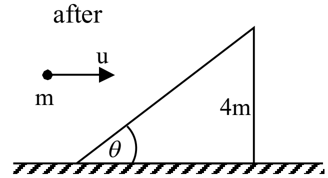
5.(D) The component of velocity along the inclined plane must remain unchanged.

$\Rightarrow \quad V_f \sin 30 = \sqrt{3}\cos 30 \quad \Rightarrow \quad V_f = 3m/s$



after before

6.(A) Parallel to inclined plane, $u\cos\theta = v_1\sin\theta \quad \Rightarrow \quad v_1 = \dfrac{u}{\sqrt{3}}$



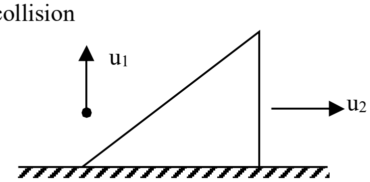
Before collision

Along horizontal $mu = 4mv_2 \quad \Rightarrow \quad v_2 = \dfrac{u}{4}$

Along common normal $v_2\sin\theta + v_1\cos\theta = eu\sin\theta$

$\Rightarrow \quad \dfrac{u}{4}\cdot\dfrac{\sqrt{3}}{2} + \dfrac{u}{\sqrt{3}}\dfrac{1}{2} = eu\dfrac{\sqrt{3}}{2} \quad \Rightarrow \quad e = \dfrac{7}{12}$

After collision



7.(C) In equilibrium, $K\cdot\dfrac{1}{100} = 2g \Rightarrow K = 2000\ N/m$ and to lift 3 kg, elongation in spring should be $\dfrac{3g}{K} = 15\ cm$.

Let 2 kg is compressed by $x \Rightarrow \dfrac{1}{2}K(0.01+x)^2 = 2g(0.01+x+0.015) + \dfrac{1}{2}K(0.015)^2$

$\Rightarrow \quad 1000\left[x^2 + 0.0001 + 0.02x\right] = 20(x+0.025) + 0.225 \quad \Rightarrow \quad x^2 = 625\times10^{-6} \Rightarrow x = 2.5\ cm$

8.(D) $\vec{a}_{cm} = \vec{g} = \dfrac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$ or $\vec{a}_2 = \dfrac{(m_1 + m_2)\vec{g} - m_1\vec{a}_1}{m_2}$

9.(A) Fore = $\dfrac{m(v_f - V_i)}{\Delta t} = \dfrac{m\left(6\hat{i} + 4\hat{j} + 5\hat{k} - \hat{i} + 2\hat{j}\right)}{.1} = \dfrac{15\left(5\hat{i} + 6\hat{j} + 5\hat{k}\right)}{.1} = 150\ \left(5\hat{i} + 6\hat{j} + 5\hat{k}\right)$

10.(D) Since the cloth is sliding under the dishes, frictional force acting on dishes is kinetic friction. Hence magnitude of this force is fixed (i.e. it is independent of velocity with which cloth is pulled). Hence the momentum imparted by cloth to dishes is proportional to time alone. The faster you pull the cloth, the lesser momentum you impart to the dishes.

11.(BC) Resolve the initial and final velocities parallel and perpendicular to the ground. Since the ground is frictionless, the parallel component will be conserved. Also, perpendicular component becomes e times in magnitude, after collision.

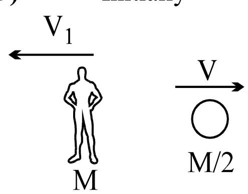
12.(ABC) $v_1 = \dfrac{m_1 - m_2}{m_1 + m_2}u_1$; $v_2 = \dfrac{2m_2u_1}{m_1 + m_2}$

13.(AC) Maximum possible velocity of ball occurs if e = 1. In this case, if v denotes the velocity of ball after collision,

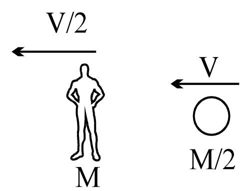
$e = 1 = \dfrac{v - u_1}{u_0 + u_1}$; This will give $v = u_0 + 2u_1$

Also, work done by racket = change in K.E. of the ball. This gives (C).

14.(ACD) Initially



After collision



$MV_1 = MV/2$

$V_1 = V/2$

$MV/2 + MV/2 = 3MV'/2$

$V' = 2V/3$

$\therefore$ Impulse = $M\left(\dfrac{2V}{3} - \dfrac{V}{2}\right) = \dfrac{MV}{6}$

Time = $\dfrac{D}{V} = \dfrac{D + \dfrac{V}{2}\left(\dfrac{D}{V}\right)}{V - (V/2)} = \dfrac{D}{V} + \dfrac{3D}{V} = \dfrac{4D}{V}$

15.(C) Since the ball is moving along the inclined plane after collision, we can say that the normal component of their relative velocity has become zero after collision. Hence (A) is correct.

During collision, wedge applies a force on the ball along its normal .Hence linear momentum of ball can be conserved only along the inclined plane.

Momentum of (ball + wedge) can be conserved only along horizontal direction as an impulsive normal reaction acts on the wedge from ground.

16.(AC) $8 \sin\theta = v \cos\theta$

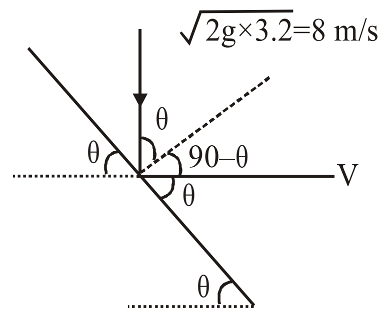
$$\frac{8\cos\theta}{2} = v\sin\theta$$

$$2 \tan\theta = \cot\theta \Rightarrow \tan\theta = \frac{1}{\sqrt{2}}$$

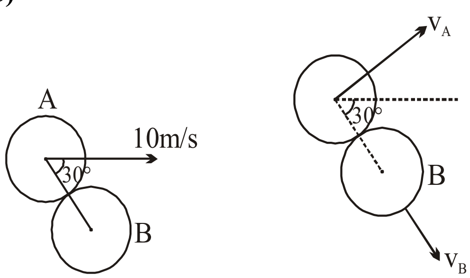
$$v = \frac{8}{\sqrt{2}} = 4\sqrt{2} \text{ m/s}$$

$$\Delta k = \frac{1}{2}\times 1\left[\left(4\sqrt{2}\right)^2 - 8^2\right] = -16 \text{ J}$$

Projectile never travels vertically downward.



17.(ABC)



$v_B = 10 \cos 30°\ 5\sqrt{3}$ and $v_A = 10 \sin 30° = 5$

18.(ACD) Since the pulley is frictionless, string will not be able to exert any tangential force on the pulley. Hence pulley will not rotate. Rest can be solved by energy conservation.

19.(AD) As particles stick after collision, so their initial momentum are the impulses imparted to 10 *m*.

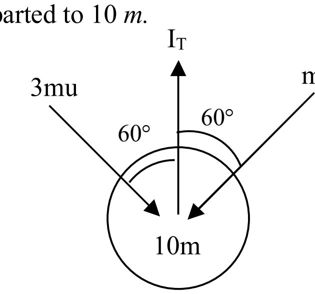
Along vertical net impulse to 10*m* is zero

$$\Rightarrow \quad I_T = mu\cos 60° + 3mu\cos 60° = 2mu$$

Along horizontal, $3mu\sin 60° - mu\sin 60° = (10+1+1)mv$

(v : velocity of combined mass just after the collision) $\Rightarrow v = \frac{\sqrt{3}u}{12}$

$$\text{Loss in energy} = \frac{1}{2}m(3u)^2 + \frac{1}{2}mu^2 - \frac{1}{2}(12m)\left(\frac{\sqrt{3}u}{12}\right)^2 = \frac{39mu^2}{8}$$



Impulse diagram of 10m

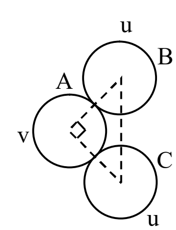
20.(BCD) At the time of impact, angle between the line following the centre of A , B and A , C is 90°

Net impulse on A = Change in momentum = *m*V.

By symmetry (C) is also correct.

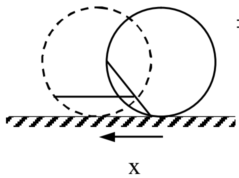
Let u : velocity of B and C after collision. $\Rightarrow$ Along the initial line of motion of A

$$mV = 2mu\cos 45° \Rightarrow u = V/\sqrt{2}$$



Initial $KE = \frac{1}{2}mV^2$ Final $KE = 2\left\{\frac{1}{2}mu^2\right\} = 2\left\{\frac{1}{2}m\left(\frac{V}{\sqrt{2}}\right)^2\right\} = \frac{1}{2}mV^2$ = initial KE

21.(B)



$\Rightarrow$ $x =$ displacement = distance of centre of mass from the centre of initial position of the sphere

$$= \frac{M(0) + M(R/2)}{M + M} = \frac{R}{2}$$

22.(ABC) From momentum conservation

$mu = mv\cos 30 + mv\cos 30$; $v = \frac{u}{\sqrt{3}}$

So, choice (C) is correct

For an oblique collision, we have to take components along normal *i.e.*, along *AB* for sphere *A* and *B*.

$v_B - v_A = e(u_A - u_B)$; $v - 0 = e\,[u\cos 30 - 0]$; $v = eu \times \frac{\sqrt{3}}{2}$; $v = e.v\sqrt{3}.\frac{\sqrt{3}}{2}$; $e = \frac{2}{3}$

So, choice (A) is correct. Also, loss of kinetic energy

$$\Delta K = \frac{1}{2}mu^2 - 2\left(\frac{1}{2}mv^2\right) = \frac{1}{2}mu^2 - 2\frac{1}{2}m\left[\left(\frac{u}{\sqrt{3}}\right)^2\right] = \frac{1}{6}mu^2$$

23.(ABCD)

The velocity of bob just before the impact is $v = \sqrt{2gl}$ along the horizontal direction

From momentum conservation

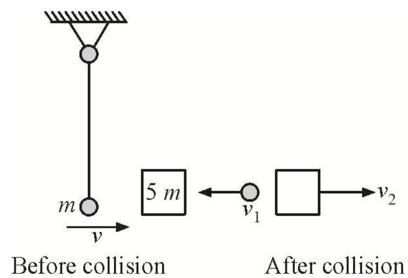
$mv = -mv_1 + 5mv_2$

From coefficient of restitution equation, $1 = \frac{v_1 + v_2}{v} \Rightarrow v_1 + v_2 = v$

Solving above equations, we get ; $v_1 = \frac{2v}{3}, v_2 = \frac{v}{3}$

For tension in string ; $T - mg = \frac{mv_1^2}{l} \Rightarrow T = \frac{17}{9}mg$

$T - mg = \frac{mv^2}{l}$, $v_2 = \frac{\sqrt{2gl}}{3}$ (T = 3 *mg*)

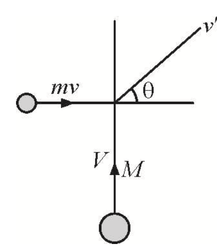


(i) Let the maximum height attained by the bob be *h*, then $\frac{mv_1^2}{2} = mgh \Rightarrow h = \frac{4l}{9}$

24.(ABC)

(A) $mv = (M + v)V'\cos\theta$

(B) $Mv = (m + M)V'\sin\theta$; $(m + M)V' = \sqrt{(mv)^2 + (MV)^2}$



(C) Initial kinetic is ; $k_l = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$

Final *K.E.* is $k_f = \frac{1}{2}(m + M)v'^2$

Decrease in $K.E. = K_l - k_f$; $\Delta k = \frac{mM}{2(m + M)}(v^2 + V^2)$

Fraction of initial kinetic transformed into heat is $\frac{\Delta k}{k_l} = \frac{mM}{m + M}\left(\frac{v^2 + V^2}{mv^2 + MV^2}\right)$

25.(AC) Area of $(a-t)$ curve $= 32\ ms^{-1} = V_f - V_i$; $V_f = 32 + V_i = 32 + 6 = 38\ ms^{-1}$

Work done by all forces $= \Delta KE$; $= \frac{1}{2}m(V_f^2 - V_i^2) = \frac{1}{2}(38^2 - 6^2) = 704\ J$

Work done by conservation forces

$U_i - U_f = 320\ J$

Work done by external forces= 704 – 320 = 384 J

26.(AC) The spring is compressed by x

Block will not return if $\mu mg \geq Kx$

So, $x_{\max} = \frac{\mu mg}{K} = \frac{(0.3)(1)(10)}{10} = 0.30\ m$

Work done against friction $= E_i - E_f$

$$\mu mg(x+2) = \frac{1}{2}mv_0^2 - \frac{1}{2}Kx^2 \quad ; \quad (0.3)(1)(10)(0.3+2) = \left(\frac{1}{2}\right)(1)v_0^2 - \left(\frac{1}{2}\right)(10)(0.3)^2$$

On solving, $v_0 = 3.8\ m/s$

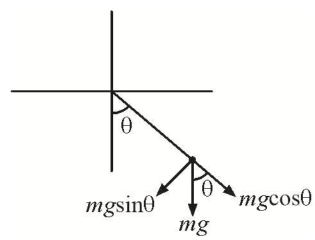
27.(AC) In case of both A and C they are path independent.

Now consider B $\int y^3 dx + xy^2 dx$ it dependent on now 'y' is related to 'x'. So case B fails same with D.

28.(BCD)

Work is said to be done in a frame any when the point of application of force undergo displacement.

29.(ABC) Between A and B



$mgl\cos\theta = \frac{1}{2}mv_B^2$; $V_B = \sqrt{2gL\cos\theta}$

$a_r = 2g\cos\theta$; $a_t = g\sin\theta$

Now, at B ; $T_B - mg\cos\theta = \frac{mv_B^2}{L}$

Put $V_B \Rightarrow T_B = 3mg\cos\theta$

$$\tan(90-\theta) = \frac{a_t}{a_r} = \frac{1}{2}\tan\theta \quad \Rightarrow \quad \tan\theta = \sqrt{2} \quad \Rightarrow \quad \cos\theta = \frac{1}{\sqrt{3}}$$

30.(ACD) Applying conservation of total energy

$$\frac{1}{2}mu^2 + mga(1-\cos\theta) = \frac{1}{2}mv^2 \quad ; \quad mg\cos\theta - N = \frac{mv^2}{a}$$

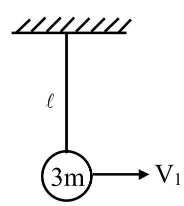
For particle to lose contact $N = 0$

$v^2 = ag\cos\theta$; $u^2 + ga(2 - 3\cos\theta) = 0$

31-33. 31.(B) 32.(B) 33.(D)

Let V_1 be speed of combined mass just after collision.

From COM in horizontal direction.



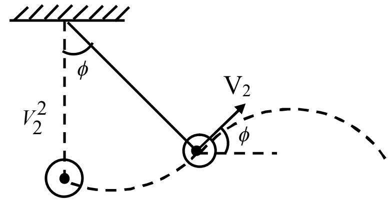
$$2m\ V\ cos\ 45° = 3m\,v_1 \left[v_1 = \sqrt{5\ g\ell}\right] \quad \Rightarrow \quad V_{min} = 3\sqrt{\frac{5g\ell}{2}}$$

At $\phi = 60°$ Let velocity = V_2.

$$3mg\,(1-\cos\phi)\,\ell = \frac{1}{2}(3m)V_1^2 - \frac{1}{2}(3m)V_2^2 \quad \Rightarrow \quad V_2 = 2\sqrt{g\ell}$$

Hence velocity at highest point = $V_2 \cos\phi = 2\sqrt{g\ell} \times \frac{1}{2} = \sqrt{g\ell}$

Maximum height = $\ell\,(1-\cos\phi) + \dfrac{V_2^2 \sin^2\phi}{2g} = 2\ell$



34-35. 34.(D) 35.(B)

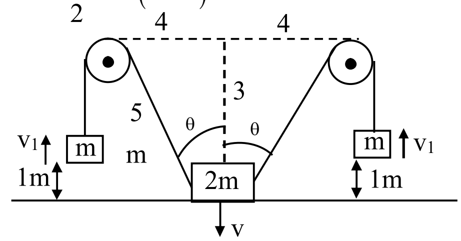
Let velocity of $2m$ and m be V and V_1 respectively then $V\cos\theta = V_1$ (using constraint relation)

$\Rightarrow \quad V_1 = 0.6\,V$

Using conservation of energy $(2m)(10)(3) = 2(m)(10)1 + \frac{1}{2}(2m)V^2 + 2 \times \frac{1}{2} \times m(0.6V)^2$

$\Rightarrow \quad V = 10\sqrt{\frac{5}{17}}\ m/s$ and $V_1 = 0.6V = 6\sqrt{\frac{5}{17}}\ m/s$

$$H_{max} = 1 + \frac{V_1^2}{2g} = 1 + \frac{36 \times \frac{5}{17}}{2 \times 10} = 1.53\ m$$



36. [A – s ; B – r ; C – q ; D – p]

A corresponds to the case where velocities are exchanged. This matches with S.

B corresponds to a perfectly inelastic collision. This matches with R as the putty is expected to be perfectly inelastic.

37. [A – q ; B – r s ; C – q p ; D – r p]

Final common velocity= 4 m/s (from cons. of momentum)

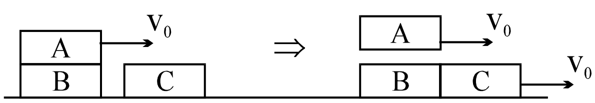
As KE of 1 kg block decreases, work done by friction on it is –ve.

Similarly, work done by friction on 2 is +ve

Total work done by friction = change in KE (2 kg + 1 kg)

$$= \frac{1}{2} \times 1 \times 6^2 + \frac{1}{2} \times 2 \times 3^2 - \frac{1}{2}(1+2)(4)^2 = 27 - 24 = 3\text{J}$$

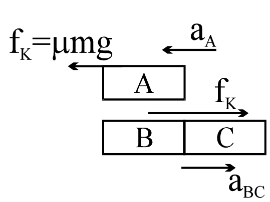
38.(1)



Form linear momentum conservative (for collision B & C)

$$mv_0 = 2mv \Rightarrow v = \frac{v_0}{2}$$

FBD



$a_A = \mu g \leftarrow$, $a_{BC} = \dfrac{\mu g}{2} \rightarrow$

$$\vec{a}_{A/B} = \frac{3\mu g}{2} \leftarrow \quad v_{A/B}^2 = U_{A/B}^2 + 2a_{A/B}\, S_{A/B} \xrightarrow[+]{}$$

$$0 = \left(\frac{v_0}{2}\right)^2 + x\left(-\frac{3\mu g}{2}\right) L$$

$$L = \frac{v_0{}^2}{12\mu g} = 1\text{m}$$

39.(2) his happens of after collision both ball and inclined plane have same horizontal velocities say V_2, say V_1 be initial velocity of ball V_y be vertical velocity of ball after collision, m_1 – mass of ball, m_2 – mass of inclined plane.

Conservation of linear momentum in horizontal direction

$$\boxed{m_1V_1 = (m_1 + m_2)V_2} \quad \text{... (1)}$$

Coefficient of restitution = 1

$$1 = -\frac{[V_2 \sin\theta - (V_2 \sin\theta - V_y \cos\theta)]}{0 - V_1 \sin\theta}$$

$$\boxed{V_y \cos\theta = V_1 \sin\theta} \quad \text{... (2)}$$

Component of velocity of ball along the inclined plane remain same

$$\boxed{V_1 \cos\theta = V_2 \cos\theta + V_y \sin\theta} \quad \text{... (3)}$$

$$\left.\begin{aligned} V_y \sin\theta &= (V_1 - V_2)\cos\theta \\ V_y \cos\theta &= V_1 \sin\theta \end{aligned}\right] \text{using (2) \& (3)}$$

$$\text{Tan}^2\theta = \frac{V_1 - V_2}{V_1}$$

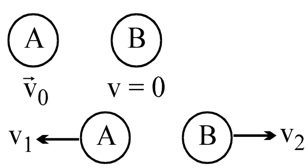
$V_2 = V_1(1 - \tan^2\theta)$ ... (4)

$(m_1 + m_2)\, V_2 = m_1V_1$ ... (1)

Divide $M_1 + m_2 = \dfrac{m_1}{1 - \tan^2\theta}$

$\dfrac{m_1}{m_2} = \cot^2\theta - 1;\quad \cot^2 30° - 1 = 2$ Ans.

40.(1) At the moment of collision



After collision

$0.25\, v_0 = -0.25\, v_1 + 0.5\, v_2$ or $2v_2 - v_1 = v_0$ ... (1)

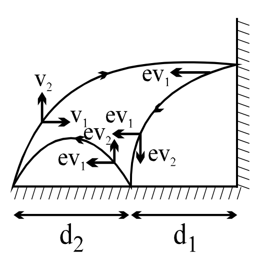
As collision is elastic,

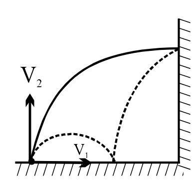
$e = 1 = \dfrac{v_2 + v_1}{v_0} \quad \Rightarrow \quad v_2 + v_1 = v_0$ ... (2)

$\therefore \quad v_2 = \dfrac{2}{3}v_0 \quad \therefore \quad v_1 = \dfrac{v_0}{3} = 1 \quad \therefore$ Velocity A is 1 m/s backward

41.(2) $(d_1 + d_2) = v_1 \left(\dfrac{v_2}{g}\right)$

$d_1 = (ev_1)\left(\dfrac{v_2}{g}\right)$





$d_2 = ev_1\left[\left(\frac{2ev_2}{g}\right)\right]$

$d_2 = 2e^2\left[\frac{v_1v_2}{g}\right]$

$d_2 = 2ed_1$; $d_1 + d_2 = \frac{d_1}{e}$ $d_1 + 2ed_1 = \frac{d_1}{e}$ $1 + 2e = \frac{1}{e}$

Solving $e = \frac{1}{2}$; Therefore, $1/e = 2$

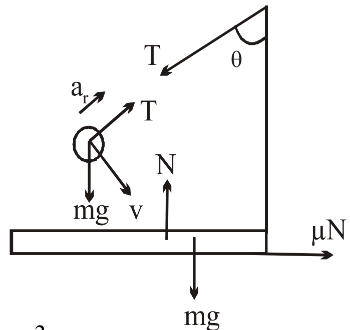
42. (3) $mg\ell\cos\theta = \frac{1}{2}mv^2 - 0$ ... (1)

$T - mg\cos\theta = \frac{mv^2}{\ell}$ ... (2)

$T\sin\theta \le \mu N$ ... (3)

$T\cos\theta + Mg = N$ ... (4)

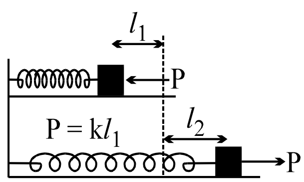
On solving $\mu \ge \frac{\sin 2\theta}{2\left[\frac{M}{3m} + \cos^2\theta\right]}$



RHS is maximum when θ = 45° ; $\mu \ge \frac{1}{\frac{2M}{3m}+1} \simeq \frac{3m}{2M} = 3\times10^{-3}$

43.(3) $P = kl_1$

$P(l_1 + l_2) = \left(\frac{1}{2}kl_2^2 + 0\right) - \left(\frac{1}{2}kl_1^2 + 0\right)$



$kl_1^2 + kl_1l_2 = \frac{k}{2}\left(l_2^2 - l_1^2\right)$

$2l_1^2 + 2l_1l_2 - l_2^2 + l_1^2 = 0$

$3l_1^2 + 2l_1l_2 - l_2^2 = 0$; $l_2 = 3l_1$

44.(6) $(1 + 3)v = (1)(8) + (3)(4) = 20$; $v = 5$ *m/sec*

For block *A*, $W_f = \frac{1}{2}(1)(5^2 - 8^2) = -\frac{39}{2}J$

For block *B*, $W_f = \frac{1}{2}(3)(5^2 - 4^2) = +\frac{27}{2}J$

Net work done by friction = – 6*J*

45.(10) Loss in gravitational = gain in *KE* + gain in elastic potential energy + work done against friction

$mgx\sin 53 = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2 + (\mu mg\cos 53)x$ …(1)

$kx + \mu mg\cos\theta = mg\sin\theta$ …(2)

Solving (1) and (2)

46.(5) $T - mg\sin\theta = \frac{mv^2}{R}$; $4mg - mg\sin 30 = \frac{m(v_0^2 + 2gl\sin 30)}{l}$; $v_0 = \sqrt{\frac{5g}{2}}$

47.(24) FBD of the block, ; $f_L = 6N,\ F_{pseudo} = 4N$ $\therefore$ $f = 4N$

Acceleration of the block with respect to observes $= 2 - 5 = -3\ m/s^2$

$\therefore$ Displacement of the block w.r.t observes $= \frac{1}{2} \times -3 \times 4 = -6m$

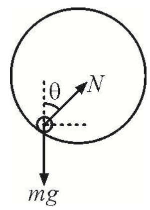
$\therefore$ Work done by friction w.r.t observes $= -24\ Joule$

48.(2) Free body diagram is:

$N\cos\theta = mg$

$N\sin\theta = m(2g)$

$\Rightarrow$ $\tan\theta = 2$ $\Rightarrow$ $\theta = \tan^{-1}(2)$



$\therefore$ Maximum possible angular displacement

$= 2\theta = 2\tan^{-1}(2)$

49.(5) $(V_{cm})_x = \dfrac{3 \times (-5\cos 37°) + 5 \times 0}{8} = -1.5\ m/s$

$(V_{cm})_y = \dfrac{3 \times (-5\sin 37°) + 5 \times 5}{8} = 2\ m/s$

$\overrightarrow{V_{cm}} = (-1.5\hat{i} + 2\hat{j})\ m/s$ $\because$ Collision at origin hence initial position of C.M. is $\vec{r_i} = 0$

$\therefore$ $(\vec{r}_{cm})_f = (\vec{r}_{cm})_i + \vec{V}_{cm}\,t = -3\hat{i} + 4\hat{j}$ $\therefore$ $\left|(\vec{r}_{cm})_f\right| = \sqrt{9+16} = 5m$

50.(2) Force F on plate = Force exerted by dust particles
= Force on dust particles by the plate
= Rate of change of momentum of dust particles
= Mass of dust particles striking the plate per
Unit time × change in velocity of dust particles $= A(v+u)\rho \times (v+u) = A\rho(v+u)^2$

51.(4) When simple pendulum released from position A strikes the wall with velocity v then by conservation of mechanical energy.

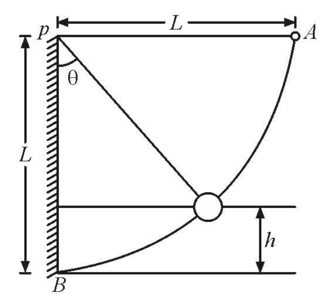
$mgL + 0 = \frac{L}{2}mv^2 + 0,\ i.e.\ v = \sqrt{2gL}$

Now as coefficient of restitution is e so, speed of pendulum after first collision will be

$v_1 = ev = e\sqrt{2gL}$

Now after completing oscillation in accordance with conservation of mechanical energy it will strike the wall with same velocity and so its velocity after second collision will be

$v_2 = ev_1 = e(e\sqrt{2gL}) = e^2\sqrt{2gL}$



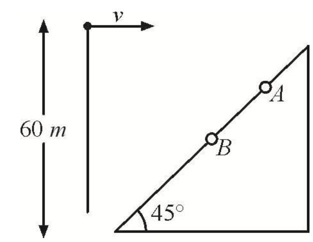
So the velocity of the pendulum after n collision will be $v_n = e^n v = e^n\sqrt{2gL}$

Now if it rises to a height h, by conservation of mechanical energy

$\frac{1}{2}m(v_n)^2 = mgh,\ i.e.,\ \frac{1}{2}e^{2n}\,2gL = gh$

or, $e^{2n} = \frac{h}{L} = \frac{L(1-\cos\theta)}{L}$ or, $\left(\frac{2}{\sqrt{5}}\right)^{2n} = 1-\cos\theta\left[\text{as } e = \frac{2}{\sqrt{5}}\right]$ or $\left(\frac{4}{5}\right)^{n} = 1-\cos\theta$

Now for θ to be lesser than 60, $\cos\theta > \left(\frac{1}{2}\right)$



i.e. $1-\cos\theta < \frac{1}{2}$

So, $\left(\frac{4}{5}\right)^{n} < \frac{1}{2}$ or, $\left(\frac{5}{4}\right)^{n} > 2$ or $n(\log 10 - 3\log 2) > \log 2$

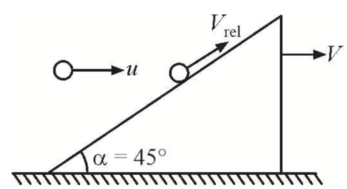
or $n > \frac{0.301}{0.097}$ [as log10 = 1 and log2 = 0.3010] or $n > 3.1$

As n (number of collisions) must be integer so for $\theta < 60°$, $n = 4$

52.(2) Let V_{rel} be the final velocity of the ball w.r.t. wedge and V be the final velocity of the wedge w.r.t. ground. Now, velocity of ball w.r.t. ground

Horizontal component $= V_x = V_{\text{rel}}.\cos\alpha + V$

Vertical component $= V_y = V_{\text{rel}}\sin\alpha$



COM in horizontal direction gives

$mu = m(V_{\text{rel}}\cos\alpha + V) + MV$ ...(1)

Since velocity of ball along wedge remains constant

$\therefore$ $u\cos\alpha = V_{\text{rel}} + V\cos\alpha$ ...(2)

Solving (1) and (2) we get ; $V = \frac{mu\sin^2\alpha}{M + m\sin^2\alpha} = 2\ m/s$

53.(4) Let velocity of I ball and II ball after collision be v_1 and v_2 ; $v_2 - v_1 = 0.5\times 10$

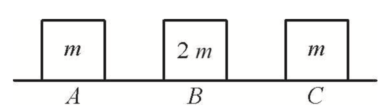
$mv_2 + mv_1 = m\times 10$ $\Rightarrow$ $v_2 + v_1 = 10$

Solving equation (1) and (2) $v_1 = 2.5\ m/s$, $v_2 = 7.5\ m/s$

Ball II after moving 10 m collides with ball III elastically and stops. But ball I moves towards ball II. Time taken for second collisions between ball 1 and 2 ; $\frac{10}{2.5} = 4$ sec

54.(4) After elastic collision

$V_A^1 = \left(\frac{m-2m}{m+2m}\right)9 = -3\ m/s$



By conservation of linear momentum after all collisions

$m(9) = m(-3) + 3m(V_e)$ or $V_e = 4\ m/s$

55.(5) Velocity of first block before collision

$v_1^2 = 1^2 - 2(2)\times 0.16 = 1 - 0.64$; $v_1 = 0.6\ m/s$

By conservation of momentum ; $2\times 0.6 = 2v_1 + 4v_2$

Also $v_2 - v_1 = v_1$ for elastic collision. It gives $v_2 = 0.4\ m/s$; $v_1 = -0.2\ m/s$

Now distance moved after collision $s_2 = \frac{(0.4)^2}{2\times 2}$ and $s_1 = \frac{(0.2)^2}{2\times 2}$; $s = s_1 + s_2 = 0.05\ m = 5cm.$

56.(2) Consider first collision between M and 4 M on the right, the velocities after collision are

$$V_M = -\frac{3}{5}u;\ V_{4M} = \frac{2}{5}u$$

Particle of mass M will move to the left and collide with 4 M. The velocities after collision are

$$V_M = \left(\frac{3}{5}\right)^2 u;\ V_{4M} = -\frac{2}{5}\times\frac{3}{5}u$$

So in all there are two collisions.

57.(7) $h_0 = 3 + \dfrac{u_B^2 \sin^2\theta}{2g} = 3 + \dfrac{2g(h-h_B)\sin^2 30°}{2g}$

$$4(\text{given}) = 3 + h\sin^2 30° - h_B\sin^2 30° = 3 + \frac{h}{4} - \frac{(3)}{4}\ ;\ \frac{7}{4} = \frac{h}{4} \qquad \Rightarrow \qquad h = 7m$$

58.(4) Decrease in mechanical energy = work done

Against friction $\dfrac{1}{2}m\upsilon^2 - \dfrac{1}{2}kx^2 = (\mu mg)x$

$$v = \sqrt{\frac{2\mu gx + k}{m}}$$

Putting m = 0.18 kg, x = 0.06 m, $k = 2\,Nm^{-1}$

$\mu = 01$ we get

$$\upsilon = 0.4\ m/s = \frac{4}{10}m/s \qquad \therefore \qquad N = 4$$

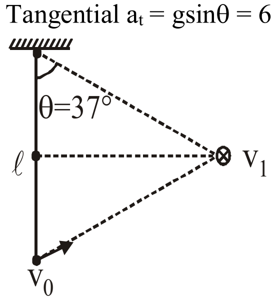
59.(6) After colliding with ground, horizontal component of velocity, *i.e.*, 10 sin30° = 5 m/s will remain unchanged while its vertical component will become zero. Collision with wall is elastic.

Hence, it will only reverse the direction of velocity of ball, magnetic will remain unchanged,

i.e., 5 m/s ; Therefore $t = \dfrac{BC+CB+BA}{V} = \dfrac{30}{5} = 6\,s$

60.(6) By Energy conservation $v_1^2 = v_0^2 - 2g\ell(1-\cos\theta) = 0$

Tangential a_t = gsinθ = 6 m / s^2



Radial a_r = 0

Resultant acceleration = $\sqrt{a_r^2 + a_t^2}$ = 6 m/s^2

ROTATIONAL MOTION

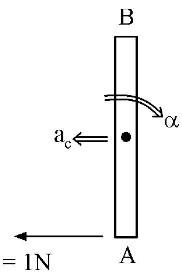
1.(C) Since all the particles on a helix are equidistant from the axis, we can use I = mR^2, where m is the total mass of wire. Length of helical wire can be found by unrolling the helix into a straight line.

2.(A) Use the formula for moment of inertia of a triangular plate about its base (I = $mh^2/6$). Note that the two diagonals of a rectangle will not be in general perpendicular to each other. Hence perpendicular axis theorem cannot be used.

3.(A) $F.\frac{\ell}{2} = \frac{m\ell^2}{12}\alpha \Rightarrow \alpha = \frac{6F}{m\ell}$

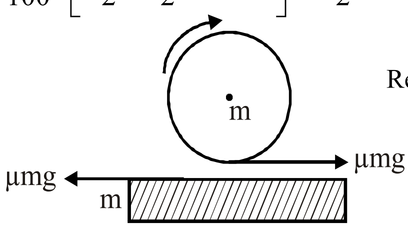
$F = ma_c \Rightarrow a_c = \frac{F}{m}$

$a_B = \frac{\ell\alpha}{2} - a_c = \frac{3F}{m} - \frac{F}{m} = 2m/s^2$ (right)



4.(B) $\frac{40}{100}\left[\frac{I\omega^2}{2} + \frac{1}{2}m(\omega r)^2\right] = \frac{I\omega^2}{2} \Rightarrow \frac{3}{5}\frac{I\omega^2}{2} = \frac{1}{5}m\omega^2 r^2$ $\quad$ $I = \frac{2}{3}mr^2$

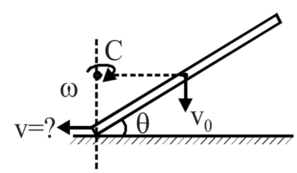
5.(D)



Relative acceleration 2μg

6.(D) $\omega\frac{L}{2}\cos\theta = v_0$

$\omega\frac{L}{2}\sin\theta = v$ $\quad$ $\therefore v = v_0\tan\theta$



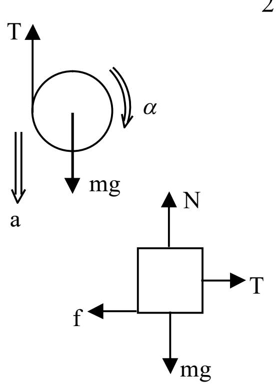
7.(B) $mg\frac{L}{2} = \frac{1}{3}mL^2\alpha$

$\therefore \quad \alpha = \frac{3g}{2L}$ and $\alpha x = g \quad \Rightarrow \quad \frac{3g}{2L}x = g \quad \therefore x = \frac{2L}{3} \quad \therefore$ Distance from B = $\frac{L}{3}$

8.(A) Impulse = $mV - (-mV_0) = m(V + V_0)$ and about centre of mass angular impulse

$= m(V + V_0)\frac{\ell}{2}\cos\theta = \frac{mV^2}{12}.\omega$ (= change in angular momentum) $\quad \Rightarrow \quad \omega = \frac{6(V+V_0)\cos\theta}{\ell}$

9.(A)



$mg - T = ma \Rightarrow mg = \frac{3}{2}ma$

$Tr = \frac{1}{2}mr^2\alpha$ $\quad$ $a = 2g/3$

$a = r\alpha$ $\quad$ $T = mg - \frac{2mg}{3} = \frac{mg}{3}$

$T = f$

$N = mg$

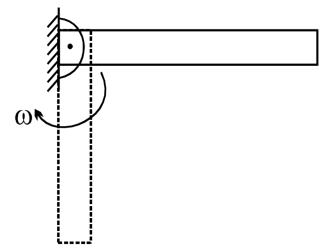
$f \le N\mu \Rightarrow \frac{mg}{3} \le mg\mu \Rightarrow \mu \ge \frac{1}{3}$

10.(C) Work done for rotation is minimum when, moment of inertia is minimum and *MI* is minimum when axis passes through the centre of mass. Let it be at a distance *x* from 0.3 *kg*.

$$\Rightarrow \qquad x = \frac{(0.3)(0)+(0.7)+(1.4)}{0.3+0.7} = 0.98$$

11.(ABCD) Work done by kinetic friction on ONE body may be positive/negative/zero. Direction of frictional force in B,C,D is correct for providing the necessary torque.

12.(BC) $I = \frac{1}{3}Ml^2$



$$\frac{1}{2}I\omega^2 = Mg \quad \Rightarrow \quad \omega = \sqrt{\frac{3g}{1}}$$

13.(AC) $N_A + f_B = 2mg$; $N_B = f_A$; $mgR + -f_A R - f_B R = 0$; $mg = f_A + f_B$

14.(BC) The angular momentum of the system is conserved. Kinetic energy will not be conserved because friction is there.

15.(ABC) Parallel Axis theorem, check the distance carefully. $I_D = I_B$ (symmetric)

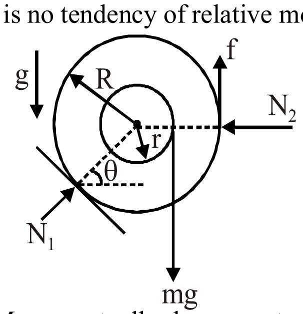
16.(BCD) Since normal is impulsive, friction will also be impulsive and it will reduce ω and give some horizontal velocity to C.M. $v \le \omega r$ friction cannot act when there is no tendency of relative motion.

17.(AC) $mgr = fR$ ... (i)

$N_1 \sin\theta + f = mg$ ... (ii)

$N_1 \cos\theta = N_2$ ... (iii)

$f \le \mu N_2$ ... (iv)



18.(AD) Due to torque of friction about CM ω eventually decreases to zero, initially there is no translation. Friction is sufficient for pure rolling therefore after sometime pure rolling begins. There is no external force in × direction therefore momentum is conserved along × direction.

19.(AD) Linear impulse = mv_0

Angular impulse = (2R/3) Linear impulse. This will give the angular speed of sphere just after collision.

Impulse of friction DURING collision is negligible.

20.(CD)

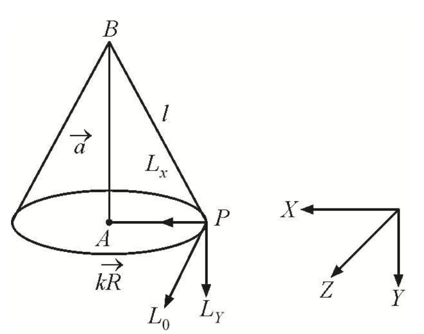
21.(AC) *AB* is $\vec{a}$

AP is $\vec{R}$

At point *P*, velocity is out of the plane

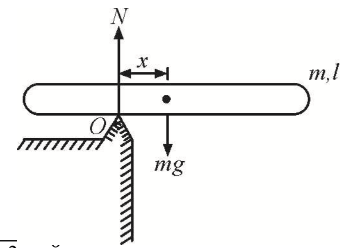
Angular momentum $\vec{L_0} = \vec{L} \times \vec{mv} = (\vec{a} + \vec{R}) \times \vec{mv}$

$$\vec{L_0} = \underbrace{(\vec{R} \times \vec{mv})}_{\text{Constant in direction magnitude}} + \underbrace{(\vec{a} \times \vec{mv})}_{\text{Constant in magnitude only}}$$



22.(AC) $I_0\alpha = \tau_0$

$$\left[\frac{ml^2}{12} + mx^2\right]\alpha = mgx \qquad \ldots(i)$$



$$\alpha = \frac{12gx}{l^2 + 12x^2}$$

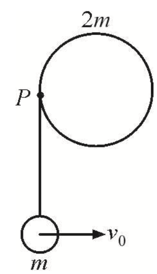
For α maximum

$$\frac{d\alpha}{dt} = 0 \quad ; \quad \frac{d\left[\frac{12gx}{l^2+12x^2}\right]}{dx} = 0 \qquad \frac{12g}{12x^2+l^2} - \frac{288gx^2}{(12x^2+l^2)^2}$$

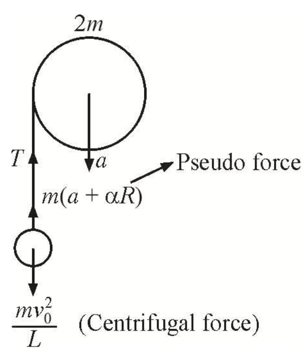
$$-\frac{12g(12x^2 - l^2)}{(12x^2+l^2)^2} = 0 \quad ; \quad 12x^2 - l^2 = 0 \quad ; \quad x = \frac{l}{2\sqrt{3}}$$

Now put x in (i) ; $\alpha = \dfrac{g\sqrt{3}}{l}$

23.(AB) Velocity of connected mass will be v_0 due to conservation of linear momentum



Drawn *FBD* with respect to *P*,



(i) $TR = I\alpha$ For $2m$ (ii) $T = 2\,ma$

(ii) $T + m(a + dR) = \dfrac{mv_0^2}{L}$ For m $\Rightarrow$ $T = \dfrac{2mv_0^2}{5l}$ and $a = \dfrac{v_0^2}{5l}$

Also after certain value of *F*, torque on cylinder will be constant hence D is corerct.

24.(ABD) $\omega_0 R = v_{com}$

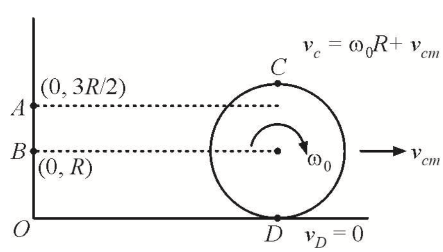
$L = mv \times R + I\omega$

$L_C = mv_{com}R(-\hat{k}) + I_{com}\omega_0(-\hat{k})$

$L_D = +mv_{com}R(-\hat{k}) + I_{com}\omega_0(-\hat{k})$

$L_C = L_D$

$L_0 = +mv_{com}R(-\hat{k}) + I_{com}\omega_0(-\hat{k})$



$L_A = +mv_{com}\frac{R}{2}(+\hat{k}) + I_{com}\omega(-\hat{k})$

$L_B = I_{com}\omega(-\hat{k})$

So L_A is minimum

$L_0 = L_C$

25.(ABCD)

$R = \frac{v^2}{a_c}$; $v = \frac{J}{M}$

$\omega = \frac{\frac{JL}{2}}{\frac{MI^2}{12}} = \frac{6J}{ML}$ $\Rightarrow$ $v_A = v + \frac{\omega L}{2} = \frac{J}{M} + \frac{3J}{M} = \frac{4J}{M}$; $a_{c_A} = \frac{\omega^2 L}{2} = \frac{18J^2}{M^2 L}$

$\Rightarrow$ $R_A = \frac{v_A^2}{a_{c_A}} = 16\frac{J^2}{M^2}\cdot\frac{M^2 L}{18J^2} = \frac{8}{9}L$; $v_B = v - \frac{\omega L}{2} = \frac{2J}{M}$ $\Rightarrow$ $R_B = \frac{4J^2}{M^2}\cdot\frac{M^2 L}{18J^2} = \frac{2}{9}L$

26.(ABCD) Basic knowledge to write angular momentum and kinetic energy of the system.

27.(BC) $x^2 = \frac{2a}{\sqrt{3}}y$; $2x = \frac{2}{\sqrt{3}}a\frac{dy}{dx}$; $\tan\theta = \sqrt{3}\frac{x}{a}$

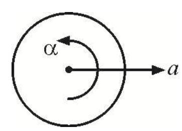
$\theta = 60°$; $a = \frac{g\sin\theta}{1+\frac{I_{cm}}{mR^2}} = \frac{g}{\sqrt{3}}$; $f = \frac{mg\sin\theta}{1+\frac{mR^2}{I_{cm}}} = \frac{mg}{2\sqrt{3}}$

28.(AD) $r = r_0 - ct$ …(i) ($\because$ distance covered by thread in time t)

By conservation of angular momentum about O,

$I_0\omega_0 = I\omega$

$(mr_0^2)\left(\frac{v_0}{r_0}\right) = mr^2\left(\frac{v}{r}\right)$



$v_0 r_0 = vr$ $\Rightarrow$ $v = \frac{v_0 r_0}{r}$ …(ii)

T at any time $= mv^2$ velocity and radius, r at that instant

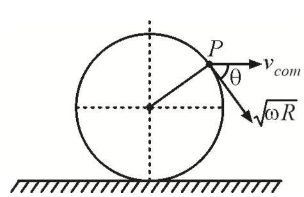
T and time $T = \frac{mv^2}{r} = \frac{m\left(\frac{v_0 r_0}{r}\right)^2}{r} = \frac{mv_0^2 r_0^2}{r^3}$

$T = \frac{mv_0^2 r_0^2}{(r_0 - ct)^3}$ …(ii)

ω at time $\omega = \frac{v}{r} = \frac{v_0 r_0}{r^2}$

$\omega = \frac{v_0 r_0}{(r_0 - ct)^2}$ [From (i) and (ii)]

29.(ABCD)



$\sqrt{v_{com}^2 + (\omega R)^2 + 2v_{com}\omega R\cos\theta}$

In frame of ωR com each point has velocity and hence same KE.

30.(BCD) $a = \alpha R$ ($\because$ no slipping)

For block, $ma = mg - T$ ...(i)

For disc, $TR - fR = I\alpha$

$$T - f = \frac{mR^2}{2} \times \frac{a}{R} \times \frac{1}{R}$$

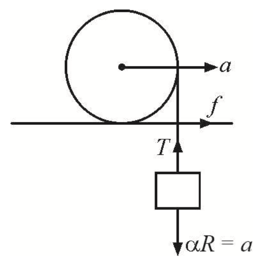
$$T - f = \frac{ma}{2} \quad \text{...(ii)}$$

And $f = ma$ ...(iii)

Put (i) and (iii) in (ii) ; $a = \frac{2}{5}g$

For block, acceleration $\rightarrow$ $-\frac{2}{5}g\hat{j}$

For disc, acceleration $\rightarrow$ $\frac{2}{5}g\hat{i}$



Acceleration of block in frame of disc

$$a_{BD} = a_B + a_D$$

$$= -\frac{2}{5}g\hat{j} - \frac{2}{5}g\hat{i}$$

From (i), $T = mg - ma$

$$T = mg - \frac{2mg}{5} \quad ; \quad T = \frac{3mg}{5}$$

31.(CD) Frictional force on disc = μmg

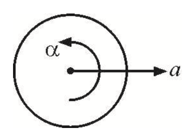
$$a = \mu g, \ \alpha = -\mu gR$$

Let the velocity and angular velocity during radian be v and ω

$\Rightarrow$ $v = u + at$ and $\omega = \omega_0 + \alpha t$ $\Rightarrow$ $v = at$ and $\omega = \omega_0 + \alpha t$

$v = \mu gt$ ...(i)

and $\omega = \omega_0 - \mu gRt$ ...(ii)



Put (i) and (ii),

$$\omega = \omega_0 - vR$$

$$\omega + vR = \omega_0$$

Now, $\omega = vR$ ($\because$ rolling) $\Rightarrow$ $2\omega = \omega_0$; $\omega = \frac{\omega_0}{2}$ $\Rightarrow$ $v = \omega R = \frac{\omega_0 R}{2}$

Time taken $= t = \frac{v}{\mu g}$ [From (i)]

$$t = \frac{\omega_0 R}{2\mu g}$$

Displacement till rolling begins $= S = ut + \frac{1}{2}at^2$; $S = 0 + \frac{1}{2} \times mg \times \frac{\omega_0^2 R^2}{4\mu^2 g^2}$; $S = \frac{\omega_0^2 R^2}{8\mu g}$

Work done by friction $= f$ is ; $= \mu mg\left(\frac{\omega_0^2 R^2}{8\mu g}\right) = \frac{\omega_0^2 R^2}{8}$

$\therefore$ Velocity v and work done by friction do not depend on value of coefficient of friction

32.(ACD) To the right of B, there is no friction therefore, no torque acts on the body

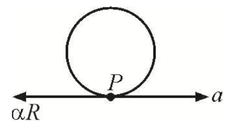
$\therefore$ The angular velocity remains constant (no angular acceleration)

Rotation KE will be constant to the right of B

To the right of B, F is still being applied; therefore the object will still undergo constant linear acceleration.

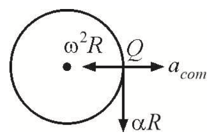
33.(ABCD)

(A) Is correct since $v = \omega R$ at P any horizontal acceleration at P will cause slipping $a = \alpha R$

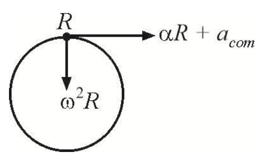


(B) If $\omega R^2 = a_{com}$

a_Q will be downward



(C) a_R can not be horizontal.



(D) Correct

34.(BC) Conservation of linear momentum

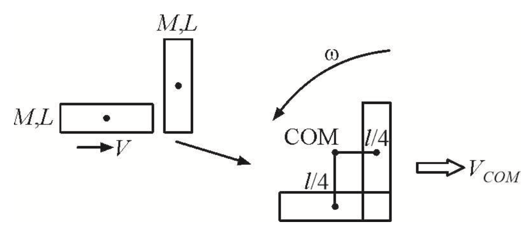
$$M_V = 2MV_{COM}$$

$$V_{CM} = \frac{V}{2}$$

Conservation of angular momentum

$$MV_{l/4} = 2MV_{COM}(0) + I_{COM}\omega$$

$$MV_{l/4} = 2\left[\frac{Ml^2}{12} + \frac{Ml^2}{8}\right]\omega \quad ; \quad \omega = \frac{3V}{5L}$$

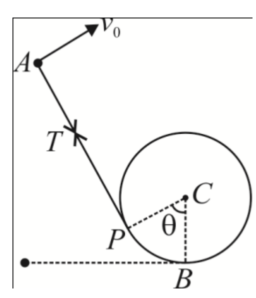


35.(D) 36.(B) 37.(B)

Let us first understand some general concepts of the problem.

Speed of the particle remains constant. Since the only force acting on the particle is tension and this force is always perpendicular to the instantaneous velocity of the particle. Hence tension does no work on the particle and by work energy theorem; speed of the particle remains constant.

Let us denote the point at which the thread touches the cylinder by P. As we can see, the speed as well as acceleration of this point is zero. Hence, at an instant, in the reference frame of this point, the particle can be taken to be performing circular motion.



(A) Torque on the particle is due to T and obviously NOT zero about B, C, or midpoint of BC. Hence answer to 'a' is none of these.

(B) $T = \frac{mv_0^2}{r}$ ($r =$ length AP).

$\because$ r continuously decreases whereas m, v_0 remain constant, T continuously increases. Torque on cylinder due to T is TR. So, this torque also continuously increases. Hence, the external torque required to keep the cylinder stationary (by balancing the torque TR) should also be increased continuously.

(C) At any instant, angular speed of segment AP is: $\omega = \frac{v_0}{r}$ where $r + R\theta = l \Rightarrow r = (l - R\theta)$

So, $\frac{d\theta}{dt} = \frac{v_0}{l - R\theta}$ ... (i)

θ goes from zero to $\theta = \frac{l}{R}$. $\therefore$ $\int_0^{l/R} (l - R\theta)d\theta = v_0 \int_0^T dt$ We can get $\boxed{T = \frac{l^2}{2Rv_0}}$

Note: $\because$ PA is always perpendicular to PC, angular speed of PC and PA are same. That is why θ on both sides of equation (i) are taken to be same.

38.(B) 39.(C)

Just after release.

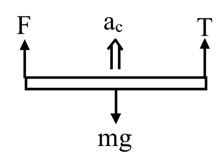
For block, $mg - T = ma$ **...(i)** ; For rod, $T\ell - mg\frac{\ell}{2} = \frac{m\ell^2}{3}\alpha$ **...(ii)**

Also $a = \ell\alpha$ **...(iii)**

$T = 5mg/8$

$\alpha = \frac{3g}{8\ell}, a = \frac{3g}{8}$

For rod $a_c = \frac{\ell}{2}\alpha = \frac{3g}{16}$ (up)



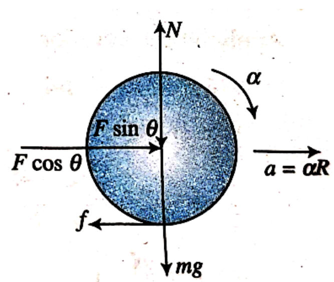
So let force exerted by hinge = F (up) then $F + T - mg = ma_c \Rightarrow F + \frac{5mg}{8} - mg = \frac{3mg}{16} \Rightarrow F = \frac{9mg}{16}$

40.(B) 41.(A) 42.(C)

$F\cos\theta - f = ma$

$fR = \frac{1}{2}mR^2\alpha \Rightarrow fR = \frac{1}{2}mR^2\frac{a}{R}$

$\Rightarrow f = \frac{ma}{2}$ .....(ii)



From (i) and (ii)

$f = \frac{F\cos\theta}{3}$

Now $a = \frac{2f}{m} = \frac{2F\cos\theta}{3m}$

$f \le f_1 \Rightarrow \frac{F\cos\theta}{3} \le \mu N$

$\Rightarrow \frac{F\cos\theta}{3} \le \mu(mg + F\sin\theta) \Rightarrow F \le \frac{3\mu mg}{\cos\theta - 3\mu\sin\theta}$

43.(D) Acceleration of the bottom of the cylinder = acceleration of the plank = $2m/s^2$ towards the right

Acceleration of the top of the cylinder = acceleration of $B = 6ms^{-2}$ towards the left.

Let linear acceleration of the cylinder be a towards the left and angular acceleration of it α in an anticlockwise sence. Writing constraint, we get

$$a + R\alpha = 6 \text{ and } R\alpha - a = 2$$

$\Rightarrow$ $\alpha = 1\ rad\ s^{-2}, a = 2m/s^2$.

44.(B) For block B ; $m_B g - T = m_B 6$ . . . (i)

Solving Eqs. (i) and (ii), $m_{eyl} : m_B = 2:1$ . . . (ii)

45.(A) The acceleration of the top part of the cylinder (thread) with respect to CM of the cylinder is

$a_{AO} = \alpha R = 1 \times 4 = 4\,m/s^2$

Length of the thread $= 20 + \frac{1}{2}(R\alpha)t^2 = 28m$

46. **[A- qrs, B-prs, C-qrs, D-prs]**

Conserve angular momentum about the hinge and use the equation for e to get angular speed of rod and speed of particle just after collision. Thereafter, you may calculate the linear momentum of rod using P = M.V_{cm}.

47. **A – s ; B – q, r ; C – s ; D – p,r**

We know that angular momentum = linear momentum perpendicular distance. So about O, angular momentum will remain constant. But about E it will keep on changing, It is maximum about E when the particle is at A, because then the perpendicular distance is maximum .

We know that angular velocity will remain constant. But about E it will keep on changing. It is minimum about E when the particle is at A, because when the perpendicular distance (r) is maximum.

48. **A – p,q ; B – p,q,r ; C – p, q ; D – p, q, r**

Since all forces on the disc pass through the point of contact with the horizontal surface, the angular momentum of the disc about the point on the ground in constant with disc is conserved. Also the angular momentum of the disc in all cases is conserved about any point on the line passing through the point of contact and parallel to the velocity of the centre of mass.

The KE of the disc is decreased in all cases due to work done by friction. From the calculation of velocity of the lowest point on disc, the direction of friction in case (A), (B) and (D) is towards the left and in case *r* it is towards the right.

The direction of frictional force cannot change in any given case.

49.(2) $\frac{Mgr}{2} = \frac{1}{2}\frac{Mr^2}{3}\omega^2$; $\omega = \sqrt{\frac{3g}{r}}$; $\sqrt{5gr}\,m \times r = \frac{Mr^2}{3} \times \sqrt{\frac{3g}{r}}$

m = 2kg

50.(6) By linear momentum conservation impulse (J) = mV.

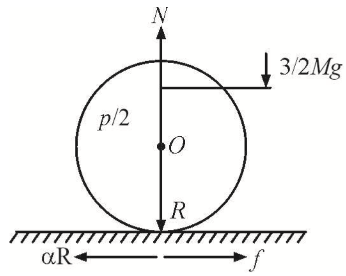
By angular momentum conservation, angular impulse = J$\frac{\ell}{2}$=Iω. So mv$\frac{\ell}{2}$ = Iω or $\omega = \frac{mv\ell}{2I} = \frac{mv\ell}{2\left(\frac{m\ell^2}{12}\right)} = \frac{6v}{\ell}$ = 6rad/s

51.(5) Time in which C.M. reaches its highest point = 1 sec. (from v = u + at, putting v = 0, u = +10, a = 10 m/sec^2) after projection angular velocity will not change as the torque of external forces is zero.

In 1 sec., the rod will rotate by an angle = $\omega t = \frac{\pi}{2} \times 1$ rad.

The rod will be vertical with point A at the lowest point. $\therefore$ $a_A = g - \omega^2 L/2 = 10 - \frac{\pi^2 4}{4 \times 2} = 5 \text{ m/sec}^2$

52.(8) For point O $\frac{3mg}{2} + mg = N$



$N = \frac{5mg}{2}$; $f = \frac{5}{2}\mu mg$

$\alpha R = a_{com}$; $a_{com} = \frac{f}{m}$

$I\alpha = \frac{3}{2}mg \times 2R - fR$

$\frac{mR^2}{2}\alpha = \frac{3}{2}mg \times 2R - \frac{5}{2}\mu mgR$; $\alpha = \frac{6g - 5\mu g}{R}$

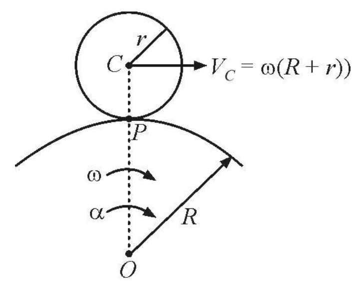
$\alpha R = \frac{f}{m} = 6g - 5\mu g$; $6mg - 5\mu g = \frac{5}{2}\mu mg$

$\mu = \frac{12}{15} = \frac{4}{5}$; $10\mu = 8$

53.(7) $\frac{R(R+r)\omega^2}{r}$; $V_C = \omega(R+r)$

Also, $V_C = \omega' r$

Where ω' is the angular velocity of cylinder about C



$\Rightarrow$ $\omega' r = \omega(R+r)$ $\Rightarrow$ $\omega' = \frac{\omega(R+r)}{r}$

a_x of point P = 0 (pure rolling) acceleration of P along Y w.r.t. C is $\omega'^2 r$ vertical upward.

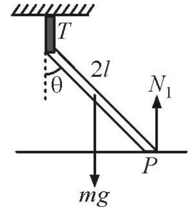
However C itself has acceleration in $Y = \omega^2(R+r)$ vertically downward

$(a_P/c)_Y + \vec{a}_C = a_P$

$\omega'^2 r - \omega^2(R+r) = a_P$

$\left\{\frac{\omega(R+r)}{r}\right\}^2 r - \omega^2(R+r) = a_P$ $\Rightarrow$ $a_P = \frac{R(R+r)\omega^2}{r}$; $\frac{7R(R+r)\omega^2}{kr}$

54.(1) Before cutting, apply equations of translational and rotational equilibrium about point P.



$N_1 + T = mg$ and $T(2l)\sin\theta = mgl\sin\theta$

$\Rightarrow$ $N_1 = mg/2$ ...(i)

After cutting, the acceleration of centre of mass will be vertically downwards.

Z about POC $= T\alpha$

$\frac{ml^2}{3}\alpha$ ...(ii)

and $\quad a_{com} = \dfrac{\alpha l}{2}$ $\quad$ …(iii)

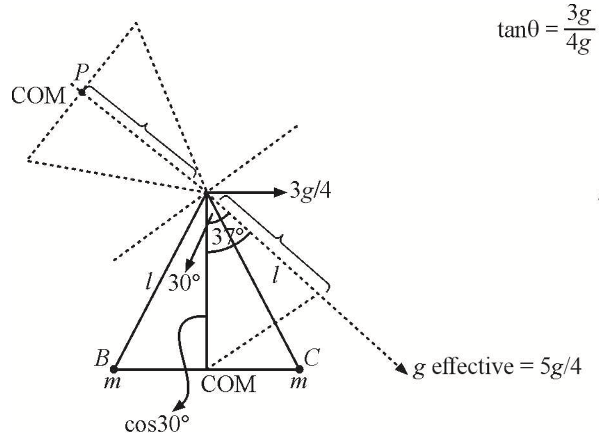
Now, along vertical

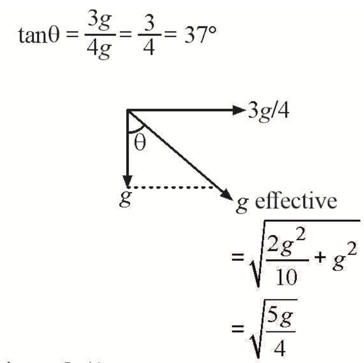
$$ma_{com} = mg - N_2 \quad \text{…(iv)}$$

Solve equation (i), (ii), (iii) and (iv)

55.(3) If the COM reaches *P*, the triangular frame will complete circle. By conservation of energy. (Assuming initial P.E. = 0)

Initial *K.E.* = find *P.E.*





$$2mg \times \frac{5}{4}(L\cos 30 \cos 37° + L\cos 30°) = \frac{1}{2} \times 2mL^2\omega^2$$

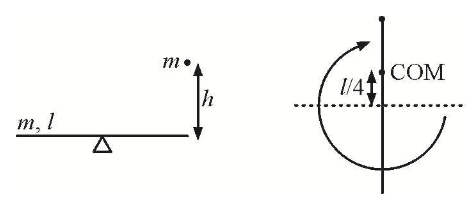
$$\frac{10 \times 10}{4}\left(\frac{\sqrt{3}}{2} \times \frac{4}{5} + \frac{\sqrt{3}}{2}\right) = 2.5\omega^2 \quad ; \quad 2\left(\sqrt{3} \times \frac{4}{5} + \sqrt{3}\right) = \omega^2$$

$$4\sqrt{3} + 5\sqrt{3} = \omega^2 \quad ; \quad \omega = (9\sqrt{3})^{1/2} = 3(3)^{1/4}$$

$$x = 3$$

56.(8) For critical case

Initial KE = P.E. at max height



$$mg\frac{l}{4} = \frac{1}{2}I_{Hinge}\omega^2$$

Angular momentum conservation about hinge

$$mv\frac{l}{2} = I\omega$$

$$m\sqrt{2gh}\,\frac{l}{2} = \left[\frac{ml^2}{12} + \frac{ml^2}{4}\right]\omega \qquad (v = \sqrt{2gh} \quad \because v^2 - u^2 = 2gh)$$

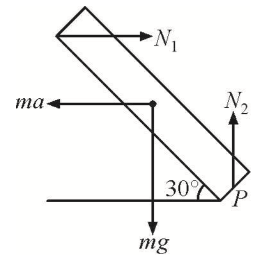
$$\frac{6}{4l}\sqrt{2gh} = \omega \quad ; \quad 2mg\frac{l}{4} = \frac{1}{2}\left[\frac{ml^2}{12} + \frac{ml^2}{4}\right]\left[\frac{36}{16l^2} \times 2gh\right]$$

$$\frac{2l}{3} = h = \frac{2}{3} \times 12 = 8$$

$$h = 8$$

57.(3) Length = $2l$

Apply pseudo force ma to left at centre of mass of rod by translational equilibrium,



$$N_1 = ma \qquad \ldots(i)$$

By rotational equilibrium about point P,

$\because \quad mal\sin 30° + mgl\cos 30° = N_1(2l)\sin 30°$

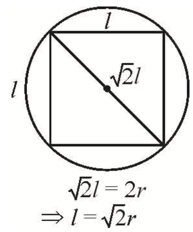
$$\frac{mal}{2} + \frac{mgl\sqrt{3}}{2} = N_1 l \qquad \ldots(ii)$$

Put (i) in (ii),

$$a = g\sqrt{3} \qquad \Rightarrow \qquad R = 3$$

58.(4) Moment of inertia of each rod

$$= \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2$$



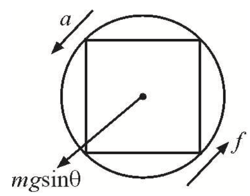
$$= \frac{2mr^2}{3} \qquad (\because\ l = \sqrt{2}r)$$

For entire object,

$$I = 4\times\left(\frac{2mr^2}{3}\right) = \frac{8mr^2}{3}$$

Now, $\quad 4ma = 4mg\sin\theta - f \qquad \ldots(i)$

and $\quad fR = I\alpha$



$\Rightarrow \quad f = \dfrac{Ia}{R^2}$

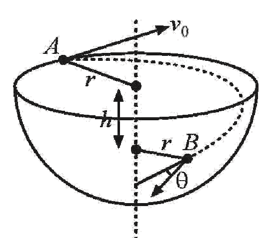
$\Rightarrow \quad f = \dfrac{8ma}{3} \qquad \ldots(ii)$

Putting (ii) in (i), $\dfrac{20ma}{3} = 4mg\sin\theta \qquad \Rightarrow \qquad a = \dfrac{12g}{20\sqrt{2}}$

Putting in (2), $\quad f = \dfrac{8m}{3}\times\dfrac{12g}{20\sqrt{2}} = \dfrac{8mg}{5\sqrt{2}}$

Now, $\quad f \le \mu N \qquad \Rightarrow \qquad f \le \mu mg\cos\theta\times 4 \qquad \Rightarrow \qquad \dfrac{8mg}{5\sqrt{2}} \le \dfrac{\mu mg}{\sqrt{2}}\times 4 \quad \Rightarrow \qquad \mu = \dfrac{4}{10}$

59.(3) Angular momentum of the particle is conserved about the vertical centre line



$mv_0r_0 = mvr\cos\theta$

where conservation of mechanical energy gives,

$\dfrac{1}{2}mv_0^2 + mgh = \dfrac{1}{2}mv^2 \qquad ; \qquad r^2 = r_0^2 - h^2 \qquad ; \qquad \cos\theta = \dfrac{1}{\sqrt{1+\dfrac{2gh}{v_0^2}}\sqrt{1-\dfrac{h^2}{r_0^2}}}$

60.(7) $M(2d\cos\alpha - d) - m\dfrac{d}{2} - 2m(d - d\cos\alpha) = 0$

$2md\cos\alpha - \dfrac{3md}{2} - 2md + 2md\cos\alpha = 0\ ;\ 4md\cos\alpha = \dfrac{7md}{2} \qquad ; \qquad \cos\alpha = \dfrac{7}{8}$

GRAVITATION

1.(C) About centre of earth, $m\sqrt{\frac{5}{6}}\,v_e R = mvr \quad (\because \Delta L = 0)$

Where v is the velocity at maximum distance r and $\frac{1}{2}m\left(\sqrt{\frac{5}{6}}v_e\right)^2 - \frac{GMm}{R} = \frac{1}{2}mv^2 = \frac{GMm}{r}$

$\Rightarrow \quad \frac{5}{6}\frac{GM}{R} - \frac{GM}{R} = \frac{5}{6}\frac{GM}{R}\cdot\frac{R^2}{r^2} - \frac{GM}{r} \qquad$ Let $\frac{r}{R} = x \quad \Rightarrow \quad x^2 - 6x + 5 = 0 \quad \Rightarrow \quad x = 5$ or 1

2.(C) $\omega = \left|\frac{V_{s_2} - V_{s_1}}{r_2 - r_2}\right| = \left|\frac{V_{s_1}/2 - V_{s_1}}{4R_{s_1} - R_{s_1}}\right| = \frac{V_{s_1}}{6R_{s_1}} = \frac{2\pi}{6T_{S_1}} \Rightarrow T^2 \propto R^3 \;\Rightarrow\; \left(\frac{1}{8}\right)^2 = \left(\frac{10^4}{R_{s_2}}\right)^3 \;\Rightarrow\; R_{s_2} = 40{,}000\ km$

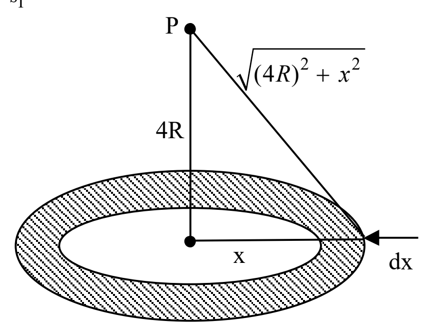
$\Rightarrow \quad R_{s_2} = 4R_{s_1}$ as $V_0 = \sqrt{\frac{GM}{R}} \;\Rightarrow\; V_{s_2} = \frac{Vs_1}{2} \qquad \Rightarrow \qquad \omega = \frac{2\pi}{6T_{s_1}} = \pi/3\ rad/hr$

3.(A) $W = \Delta V = V_\infty - V_p = -V_p$

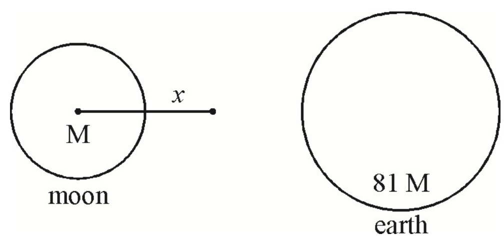
To find V_p we considering of radius x and thickness dx.

$dV_p = -\frac{GdM}{\sqrt{x^2 + 16R^2}}, \Big|\; dM = \frac{M(2\pi x dx)}{\pi(4R)^2 - \pi(3R)^2} = \frac{2Mxdx}{7R^2}$

$\Rightarrow \quad V_P = -\int \frac{GdM}{x^2 + 16R^2} = -\int_{3R}^{4R} \frac{2MGxdx}{7R^2\sqrt{x^2 + 16R^2}} = \frac{2GM}{7R}\left(4\sqrt{2} - 5\right)$



4.(A)



$\Rightarrow \frac{GM}{x^2} = \frac{G(81M)}{(60R - x)^2} \Rightarrow x = 6R$

5.(B)

6.(A) If another hemisphere(identical) is added so that it becomes a complete sphere then total intensity at both point P and Q becomes same $= I_P + I_Q$ where I_p and I_Q are intensities at P and Q respectively due to only given hemisphere

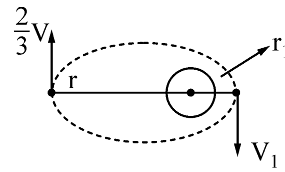
$\Rightarrow \quad I_P + I_Q$ = intensity due to complete sphere $= \frac{G(2m)}{(2R)^2} = \frac{Gm}{2R^2} \qquad \Rightarrow I_Q = \frac{Gm}{2R^2} - I_P$

7.(B) $\frac{1}{2}mu^2 - \frac{GMm}{R} = \frac{-GMm}{R + h} \qquad \Rightarrow \qquad \frac{(4000)^2}{2} - 10\left(6.4 \times 10^6\right) = -10\frac{\left(6.4 \times 10^6\right)^2}{6.4 \times 10^6 + h}$

$\Rightarrow \quad 8 - 64 = \frac{-64 \times 10^6 \times 6.4}{6.4 \times 10^6 + h} \;\Rightarrow\; 6.4 \times 10^6 \times 7 + 7h = 8 \times 6.4 \times 10^6 \;\Rightarrow\; h = \frac{64}{7} \times 10^5\ m \approx 914.3\ km$

8.(A)

9.(A) $V = \sqrt{\frac{GM}{r}}$

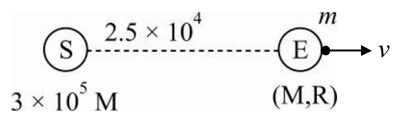


$m \times \sqrt{\frac{2}{3}}\, v \times r = mv_1 r_1$ (r_1 = min distance) **. . . . (i)**

Also $\frac{1}{2} m \times \frac{2}{3} v^2 - \frac{GMm}{r} = \frac{1}{2} MV_1^2 - \frac{GMm}{r_1}$ **. . . . (ii)**

Solving $r_1 = \frac{r}{2}$

10.(B) For earth, $Ve = \sqrt{\frac{2GM}{R}}$



For Sun + Earth,

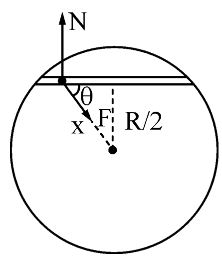
Potential energy $= \left(-\frac{GM}{R} - \frac{3\times10^5}{2.5\times10^4}\frac{GM}{R}\right)m$

$= -\frac{GMm}{R}(1+12) = -\frac{13GMm}{R}$ $\therefore$ $\frac{1}{2}mv^2 = \frac{13GMm}{R}$

$v = \sqrt{\frac{2GM}{R}}.\sqrt{13} = \sqrt{13}\, v_e$ $\therefore$ $v_e = 40.4\ km/s \simeq 42\ km/s.$

11.(BC) $N = F\sin\theta$

$= \frac{GMmx}{R^3} \times \frac{R}{2x} = \text{const.}$



$a = \frac{F\cos\theta}{m} = \frac{GMx}{R^3} \times \frac{\sqrt{x^2 - R^2/4}}{x}$

$= \frac{GM}{R^3}\sqrt{x^2 - R^2/4}$

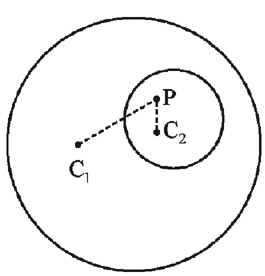
12.(AC) $V_0 = \sqrt{\frac{GM}{r}}$, $K_0 = \frac{1}{2} m \times \frac{GM}{r}$, $E = -\frac{GMm}{2R}$; $\frac{1}{2}mV_e^2 - \frac{GMm}{r} = 0$

$\frac{1}{2}mV_e^2 = \frac{GMm}{r} \Rightarrow K = \frac{GMm}{r} \Rightarrow V_e = \sqrt{\frac{2GM}{r}} = \sqrt{2}V_0 = 1.41\, V_0 \Rightarrow$ speed increases nearly 41

13.(ABC) $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{4}\right)^4 \Rightarrow T_1 : T_2 = 1:8$ $V_0 = \sqrt{\frac{GM}{r}} \Rightarrow V_1 : V_2 = \sqrt{\frac{1}{R} \div \frac{1}{4R}} = 2:1$

$L = mvr \Rightarrow L_1 = m(2v)r,\ L_2 = mv(4r) \Rightarrow L_1 : L_2 = 1:2$

14.(CD)



Field at P $= \rho G\frac{4}{3}\pi\, \overrightarrow{PC_1} - \rho G\frac{4}{3}\pi\, \overrightarrow{PC_2}$; ρ : density of massive sphere.

15.(ACD) For M

$$[M]=[h]^p[C]^q[G]^r$$

$$[M]=\left[M^1L^2T^{-1}\right]^p\left[L^1T^{-1}\right]^q\left[M^{-1}L^3T^{-2}\right]^r$$

$$[M]=[M]^{p-r}[L]^{2p+q+3r}[T]^{-p-q-2r}$$

$\therefore$ $p-r=1$ **…(i)**

$2p+q+3r=0$ **…(ii)**

$-p-q-2r=0$ **…(iii)**

On solving (i), (ii) & (iii) we get

$p=\frac{1}{2},\ r=\frac{-1}{2}$ and $q=\frac{1}{2}$ $\Rightarrow$ $[M]\propto\sqrt{h}$

$[M]\propto\sqrt{C}$

$[M]\propto\frac{1}{\sqrt{G}}$

Similarly For [L]

$p-r=0$ **…(iv)**

$2p+q+3r=1$ **…(v)**

$-p-q-2r=0$ **…(vi)**

On solving (iv), (v) & (vi)

$p=\frac{1}{2},\ q=\frac{-3}{2},\ r=\frac{1}{2}$ $\Rightarrow$ $[L]\propto\sqrt{h}$; $[L]\propto\frac{1}{C^{3/2}}$; $[L]\propto\sqrt{G}$.

16.(AD) $m_1=m,\ m_2=2m\ \Rightarrow\ r_1=\frac{2mr}{m+2m}=\frac{2r}{3}$

$$\frac{Gm(2m)}{r^2}=\frac{mV_1^2}{r_1}\Rightarrow V_1^2=\frac{2Gmr_1}{r^2}$$

$T_1=\frac{2\pi r_1}{V_1}\ \Rightarrow\ T_1^2=4\pi^2r_1^2\times\frac{r^2}{2Gmr_1}=\frac{4\pi^2r^2r_1}{2Gm}=\frac{4}{3}\frac{\pi^2r^3}{Gm}$; $T_1^2\propto r^3,\ T_1^2\propto m^{-1}$

17.(ABC)

(A) By the conservation of energy

$$0-\frac{GMm}{2R}=\frac{1}{2}mv^2-\frac{GMm}{R}$$

So, $v=\sqrt{gR}$

(B) Let escape velocity at this orbit is v_e

$$\frac{1}{2}mv_e^2-\frac{GMm}{2R}=0\ \Rightarrow\ v_e=\sqrt{\frac{GM}{R}}$$

(C) Total energy of satellite $E=-\frac{GMm}{2(R+h)}$

Total energy in the new orbit

$$E_2 = -\frac{GMm}{2(R + R/2)} \frac{GMm}{3R}$$

Total energy in the old orbit

$$E_1 = -\frac{GMm}{2.(2R)} = -\frac{GMm}{4R}$$

Energy spent = total energy of satellite in the new orbit – total energy of satellite in old orbit

Energy spent $= E_2 - E_1 = \left(-\frac{GMm}{3R}\right) - \left(-\frac{GMm}{4R}\right) = -\frac{GMm}{12R}$

18.(ABCD)

As initially the particles are at an infinite separation hence total energy is zero and by conservation of energy during motion of particles total energy will remain zero. At a separation *r* the potential energy of the particles can be given as $\frac{4Gm^2}{r}$ hence the kinetic energy magnitude must be same for total energy to be zero and by kinetic energy we can calculate the velocity of approach as no external force or torque is present in the system total angular momentum of the system will remain conserved.

19.(C) $\frac{2G\lambda}{r} \times m = \frac{mv^2}{r}$

Where $\lambda =$ mass per unit length $= \rho \times A = \rho \times \pi R^2$

$\Rightarrow \quad \frac{2G \times \rho \times \pi R^2}{r} = \frac{v^2}{r} \quad \Rightarrow \quad v = R\sqrt{2\pi G\rho}$

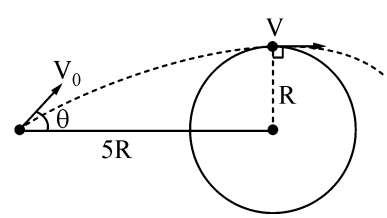
20.(B) $\int_R^{3R} \frac{2G\lambda_m}{r} dr = \frac{1}{2} mv^2 \Rightarrow v = 2R\sqrt{\pi\rho g \ln 3}$

21.(D) $T = \frac{2\pi d}{v} \Rightarrow d = RT\sqrt{\frac{G\rho}{2\pi}}$

22. **[A-p, r] [B-p, q, r, s] [C-p, q, r, s] [D-p, r, s]**

23.(5) Conservation of angular momentum

$mV_0 \sin\theta \times 5R = mVR$ ..... **(i)**



Conservation of energy

$-\frac{GMm}{5R} + \frac{1}{2} m V_0^2 = -\frac{GMm}{R} + \frac{1}{2} mV^2$ ..... **(ii)**

Solving $\theta = \sin^{-1}\left[\frac{1}{5}\sqrt{1 + \frac{8GM}{5V_0^2 R}}\right]$

24.(2) For a particle at a distance r from the center of Earth, force is given by,

$F = \frac{Gm_1 m_2}{r^2}$

Force becomes one fourth, when $r = 2R$ (R = radius of Earth)

Escape velocity, $V_e = \sqrt{\frac{2GM}{R}}$

Using conservation of energy for the given particle,

$$\frac{1}{2}mV^2 - \frac{GmM}{R} = -\frac{GmM}{2R}$$

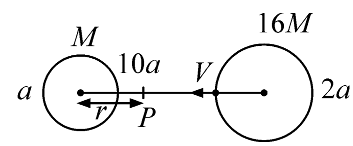
This gives,

$$V = \sqrt{\frac{GM}{R}}$$

$V_e = V\sqrt{2}$; Hence, $n = 2$

25.(5) P is neutral point

$$\frac{GM}{r^2} = \frac{G(16M)}{(10a-r)^2} \Rightarrow r = 2a$$



$$-\frac{GMm}{8a} - \frac{G16Mm}{2a} + \frac{1}{2}mV^2 = -\frac{GMm}{2a} - \frac{G16Mm}{8a}$$

$$\Rightarrow \quad V = \frac{3}{2}\sqrt{\frac{5GM}{a}} \quad \Rightarrow \quad K = 5$$

26.(1) mass of Cavity $M' = \frac{M}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$

$$F_{net} = F_{full} - F_{cavity} = \frac{GMm}{(2R)^2} - \frac{GM'm}{(2.5R)^2} = \frac{23}{100}\frac{GMm}{R^2} \quad \Rightarrow \quad K = 23, \frac{K}{23} = 1$$

27.(4) $-\frac{GMm}{3R} = \frac{1}{2}mV^2 - \frac{GMm}{R} \quad \Rightarrow \quad V = \sqrt{\frac{4GM}{3R}} \quad \Rightarrow \quad n = 4$

28.(2) $2 \times$ total energy = G.PE $\Rightarrow$ $TE = -2MJ$ At ∞, total energy = 0 $\Rightarrow$ additional energy provided = 2MJ.

29.(6) $\frac{L}{L_2} = \frac{m_1v_1r_1 + m_2v_2r_2}{m_2v_2r_2} = 1 + \frac{m_1v_1}{m_2v_2}\frac{r_1}{r_2} = 1 + 1\left(\frac{11}{2.2}\right) = 6$

30.(3) $d \propto \frac{M}{R^3} \Rightarrow d \propto \frac{M}{R^2}.\frac{1}{R} \Rightarrow d \propto g\left(\frac{g}{v_e^2}\right) \Rightarrow v_e^2 d \propto g^2$

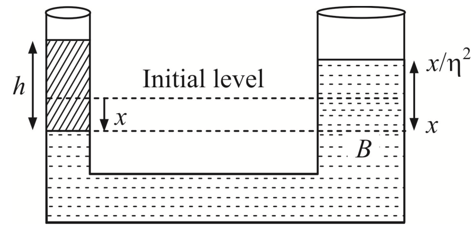
$$\frac{V_e^2}{(11)^2}\frac{2}{3} = \frac{6}{121} \Rightarrow V_e^2 = 9 \quad V_e = 3$$

LIQUIDS

1.(C) Height of water which exerts same pressure as oil is : $h\rho_w g = 5\times \rho_{oil}\times g \Rightarrow h = 5\times 0.8 = 4m$

Total height = 10 + 4 = 14 $m \Rightarrow v = \sqrt{2\times 10\times 14} = \sqrt{280}\ m/s$.

2.(B) Let mercury level drops in the left side by x mercury on right side would rise by $\frac{x}{n^2}$

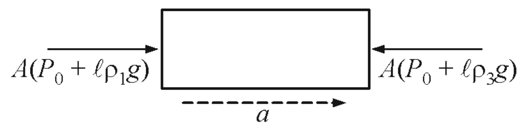


Equating pressure at A & B

$$h\rho g = \left(x + \frac{x}{n^2}\right)\rho s g \Rightarrow \frac{x}{n^2} = \frac{h}{\left(n^2+1\right)s}$$

3.(D) The whole system must have a horizontally rightward acceleration for heights of both side to be same

$\rho_1 gl = \rho_2 la + \rho_3 gl$



4.(A) ρ = density of material ;

ρ_0 = density of water when the sphere has just started sinking, the weight of the sphere = weight of water displaced

$$\Rightarrow \frac{4}{3}\pi\left(R^3 - r^3\right)\rho g = \frac{4}{3}\pi R^3\rho_0 g \Rightarrow \frac{\left(R^3 - r^3\right)}{R^3} = \frac{\rho_0}{\rho} \Rightarrow \frac{r}{R} = \frac{(7)^{1/3}}{2}$$

5.(C) $V_3 = \sqrt{2gh}$; using continuity equation at section '2' and section '3'

$$\frac{A}{2}V_2 = \frac{A}{4}V_3 \Rightarrow V_2 = \frac{1}{2}V_3 = \sqrt{\frac{hg}{2}}$$

Using Bernoullie's theorem at section '2' and at the opening end of pipe '3'

$$P_0 + \frac{1}{2}\rho V_3^2 = P_2 + \frac{1}{2}\rho V_2^2 \Rightarrow P_0 + \frac{1}{2}\rho\left(V_3^2 - V_2^2\right) = P_2 = P_0 + \frac{1}{2}\rho\left(2gh - \frac{gh}{2}\right); P_2 = P_0 + \frac{3\rho gh}{4}$$

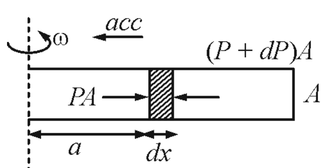
6.(B) Initially, $w - 2kx = 0$ .....**(i)** Finally, $w' - 2k\left(x + \frac{a}{2}\right) - \frac{a^3}{2}.2\sigma g = 0$ .....**(ii)**

$$w' = w + w_0 \Rightarrow w_0 = a\left(k + a^2\sigma g\right)$$

7.(A) $Mg + 2T_1\cos 45° = F_b = \left(\frac{4}{3}\pi R^3\right)\rho_w g$ $\therefore$ $T_1 = \left(\frac{\frac{4}{3}\pi R^3\rho_w g - Mg}{\sqrt{2}}\right)$

8.(C) Loss in GPE = $\frac{m}{\rho'}\rho g h_1 \Rightarrow h_1 = \frac{\rho' h}{\rho - \rho'} \Rightarrow mg\left(h + h_1\right) = \frac{m}{\rho'}\rho g h_1 \Rightarrow h_1 = \frac{\rho' h}{\rho - \rho'}$

9.(A)



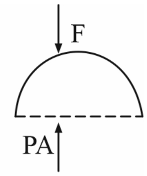
$$\int_0^P dP.A = \int_0^L \left(Adx\right)\rho\omega^2 x \Rightarrow P = \frac{1}{2}\rho\omega^2 L^2$$

$$F = PA = \frac{\rho AL}{2}\omega^2 L = \frac{M\omega^2 L}{2}$$

10.(A) $PA - F = F_B = \frac{2\pi}{3} r^3 \rho_1 g$

$(P_0 + \rho_1 gh)\pi r^2 - F = \frac{2\pi}{3} r^3 \rho_1 g$

$F = P_0 \pi r^2 + \left(h - \frac{2}{3} r\right) \pi r^2 \rho_1 g$

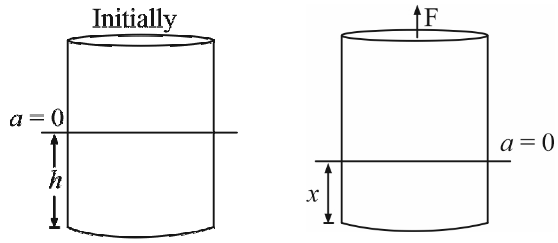


11.(B) Applying Newton's law in vertical direction

$mg = F_B$

$\Rightarrow \quad dSH \times g = \rho_w Sh \times g$

$\Rightarrow \quad h = \frac{dH}{\rho}$



Now when force F is applied, for minimum work $a = 0$ ($\because$ For $a = 0, F$ is minimum)

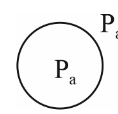
$F - mg + \rho Sxg = 0$

$W = \int F\,dx = \int (mg - \rho Sxg)\,dx = mg \int dx - \rho Sg \int x\,dx$

$W = mgh - \frac{\rho Sgh^2}{2} = (\rho Sh)\,gh - \frac{\rho Sgh^2}{2} = \frac{\rho Sgh^2}{2} = \frac{\rho Sg}{2}\left(\frac{dH}{\rho}\right)^2 = \frac{Sgd^2H^2}{2\rho}$

12.(B) Inside pressure must be $\frac{4T}{r}$ greater than outside pressure in bubble. This excess pressure is provided by charge on bubble.

$\frac{4T}{r} = \frac{\sigma^2}{2\varepsilon_0}$; $\frac{4T}{r} = \frac{Q^2}{16\pi^2 r^4 \times 2\varepsilon_0}$; $\ldots\left[\sigma = \frac{Q}{4\pi r^2}\right]$



$Q = 8\pi r\sqrt{2rT\varepsilon_0}$

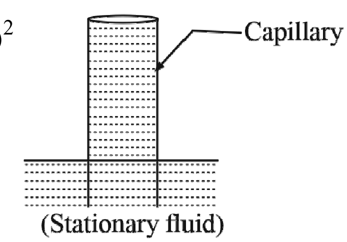
13.(C) $r = R\sin\theta$

Required force $= [2\pi r]T\sin\theta = 2\pi R\sin^2\theta T$

14.(D) Let the density of water be ρ, then the force by escaping liquid on container $= \rho S(\sqrt{2gh})^2$

$\therefore$ Acceleration of container $a = \frac{2\pi Sgh - \mu\rho vg}{\rho v} = \left(\frac{2Sh}{u} - \mu\right)g$

Now $\mu = \frac{Sh}{v}$ $\quad \therefore \; a = \frac{Sh}{v} g$



15.(ABD) $P_D = P_B$ (Pascal's Law)

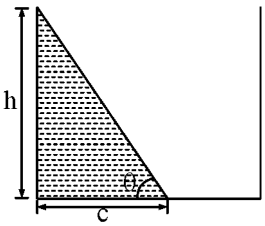
Let ℓ be the side of the square $P_B = P_D = P_C - \rho g \frac{\ell}{\sqrt{2}}$ .....**(i)**

Similarly, $P_B = P_D = P_A + \rho g \frac{\ell}{\sqrt{2}}$ .....**(ii)** $\quad \therefore \quad P_B = P_D = \frac{P_A + P_C}{2}$

16.(ACD) $\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2} \Rightarrow P_1 - P_2 = \frac{\rho}{2}\left(V_2^2 - V_1^2\right)$

But, $P_1 - P_2 = \rho gh = \frac{\rho}{2}\left(V_2^2 - V_1^2\right) \Rightarrow V_2^2 - V_1^2 = 2gh$

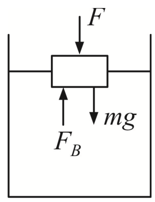
17.(AC) $\tan\theta = \frac{h}{c} = \frac{a}{g} \Rightarrow a = \frac{hg}{c}$



Maximum volume that can be retained = $\frac{1}{2} \times h \times c \times b$

And $F = \left[M + \frac{hcb\rho}{2}\right]\frac{hg}{c}$

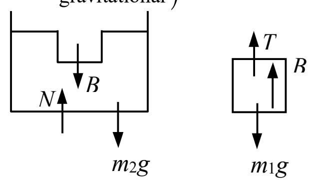
18.(ABD)



$$W_F + W_{F_B} - W_{mg} = 0$$
$$W_F = -W_{F_B} - W_{mg}$$
$$= -W_B + \Delta u_{\text{gravitational}}$$
$$= -\left(W_B - \Delta u_{\text{gravitational}}\right)$$

19.(CD) $T + B = m_1 g$

$$N = B + m_2 g$$
$$\Rightarrow \quad N = (m_1 + m)g - T$$



20.(BC) Since body is floating, Buoyant force is same in both liquids and is equal to the weight of the body.

21.(BD) In both cases $V_A = V_B$

$$\frac{P_A}{P_g} + \frac{v^2}{y} = \frac{P_B}{P_g} + \frac{v^2}{y} + h \quad ; \quad P_A > P_B \text{ in both cases.}$$

22.(AB)

23.(ABD) $F_{Buoyant} = (m_A + m_B)g;\ 2vd_{Fg} = v(d_A + d_B)g$

$d_A + d_B = 2d_F$. Therefore A, B, D

24.(AB) By equation of continuity,

$$A_1V_1 = A_2V_2$$

$$S_1 \times u = S_2 \times u\cos\theta \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{S_1}{S_2}\right) \quad \therefore \quad \sin\theta = \sqrt{1 - \frac{S_1^2}{S_2^2}}$$

Now range $R = \frac{u^2 \sin 2\theta}{g} = \frac{u^2\, 2\sin\theta\cos\theta}{g} = \frac{2u^2}{g}\frac{S_1}{S_2}\sqrt{1 - \frac{S_1^2}{S_2^2}}$

$$H_{\max} = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2}{2g}\left(1 - \frac{S_1^2}{S_2^2}\right)$$

Rate of flow volume is $(S_1 \times u)$ or $(S_2 \times u\cos\theta)$ etc.

25.(D)

26.(C) $mg = F_b$

$$a^3 \times 0.4\rho g = a^2 h\rho g \quad ; \quad h = \frac{2}{5}a = 0.4a$$

$$m = a^3 \times 0.4\rho \quad ; \quad F_{\text{net}} = a^2 x\rho g$$

$$T = 2\pi\sqrt{\frac{a^3 \times 0.4\pi}{a^2\rho g}} \quad ; \quad T = 2\pi\sqrt{\frac{2a}{5g}}$$

27.(B) Displacement must be less than submergence depth of cube.

28.(A) $v_1\rho_w g = (v-\Delta v)\rho_i g + \Delta v(\rho_m)g$

$v_2\rho_w g = (v-\Delta v)\rho_i g + \Delta v(\rho'_m)g$

$$\frac{v_1}{v_2} = \frac{(v-\Delta v)\rho_i + \Delta v\rho_m}{(v-\Delta v)\rho_i + \Delta v\rho'_m} = \frac{\left(\frac{v}{\Delta v}-1\right)\rho_i + \rho_m}{\left(\frac{v}{\Delta v}-1\right)\rho_i + \rho_m} = \frac{\left(\frac{1000}{20}-1\right)\times 0.9 + 4.9}{\left(\frac{1000}{20}-1\right)\times 0.9 + 1.9} = \frac{49}{46}$$

29.(C) Mass of cavity is more in cube A than in cube B.

30.(D) So long as cubes are floating, respective water levels do not change.

Let at $t = t_0$, cube A sinks.

$$v\rho_w g = (v-\Delta v)\rho_i g + \Delta v\rho_m g$$

v is volume of cube which is changing linearly with time at $t > t_0$.

$$v\rho_w g + N_A = (v-\Delta v)\rho_i g + \Delta v\rho_m g$$

After sinking water level decreases due to melting of ice,

$$\frac{1}{10}\frac{dv}{dt} = A\frac{dh_1}{dt}; A\text{-cross-section area of vessel}$$

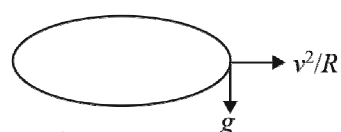
Let at $t = t'_0$, cube B sinks ; $v'\rho_w g = (v'-\Delta v)\rho_i g + \Delta v\rho'_m g$

$$\frac{1}{10}\frac{dv'}{dt} = A\frac{dh_2}{dt} \quad \because \quad \frac{dv}{dt} = \frac{dv'}{dt} \quad \Rightarrow \quad \frac{dh_1}{dt} = \frac{dh_2}{dt}$$

Final heights are same in both reach.

31.(B) $p = p_0 - pg_{\text{eff}}$

$$g_{\text{eff}} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2} \le 3g$$

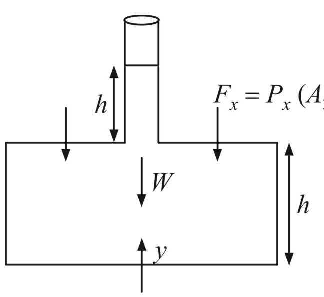


$$g^2 + \frac{v^2}{R^2} \le 8g^2 \quad ; \quad \frac{100^4}{R^2} \le 8g^2 \quad ; \quad R \ge \sqrt{\frac{100^4}{8\times 100}} = 100\sqrt{\frac{25}{2}} = \frac{500}{\sqrt{2}} = 250\sqrt{2}$$

32.(D) $P - P_0 = \Delta P = h\rho\, g_{eff} = (0.30)\,(10^3)\,(3\times 10) = 9\times 10^3\,Pa$

33.(D) Only in case $D, g_{\text{eff}} > g$

34.(A) **P → 2 ; Q → 1 ; R → 1 ; S → 3**

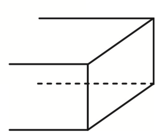


$F_x = P_x\,(A_2 - A_1) = h\rho g\,(A_2 - A_1)$

$F_y = P_y A_2 = (2h\rho g)A_2$

$F_x + W = F_y$

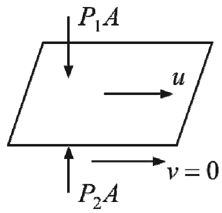
$W = F_y - F_x$ which is less than $(2h\rho g)A_2$



35.(C) **(P)** $F = P_c \times A = \frac{h\rho g}{2} \times A$

(Q) $F_B = V\rho g = (hA)\rho g$ **(R)** $F_R = \frac{\Delta P}{\Delta t} = \frac{(V\Delta t)\rho u}{\Delta t} = \rho u^2 A$

(S)

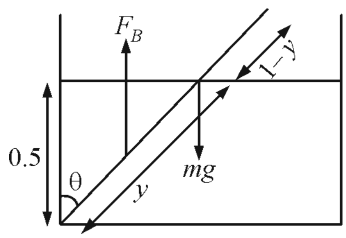


$$P_1 + \frac{1}{2}\rho u^2 = P_2 + 0$$

$$(P_2 - P_1) = \frac{1}{2}\rho u^2$$

$$F = (P_2 - P_1)A = \frac{1}{2}\rho u^2 A$$

36.(2)



$$F_B = (yA)\rho_w g$$

$$mg = 1 \times A \times \frac{\rho_w}{2} \times g$$

Balancing torque $yA\rho_w g \times \frac{y}{2}\sin\theta = 1 \times A \times \frac{\rho_w}{2} \times g \times \frac{1}{2} \times \cos\theta \Rightarrow \cos^2\theta = \frac{1}{2} \Rightarrow \frac{1}{\cos^2\theta} = 2$

37.(8) $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3 \Rightarrow R = n^{1/3}r$ or $R = 2n^{1/3}$ mm $\Rightarrow \frac{V_0^1}{V_0} = \frac{R^2}{r^2} = \frac{4n^{2/3}}{4} = 4 \Rightarrow n = 8$

38.(2) Applying Bernoulli's equation at cross-section 1 and 2.

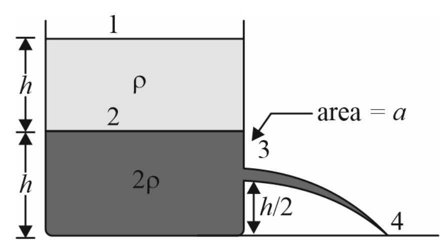
$$P_{atm} + \rho gh + 0 = P_2 + 0 + 0$$

$\Rightarrow$ $P_2 = P_{atm} + \rho gh$ …(i)

Again applying Bernoulli's equation at section 2 and 3

$P_2 + 0 + 2\rho g \times \frac{h}{2} = P_{atm} + \frac{1}{2}2\rho V^2$ …(ii)

$\Rightarrow$ $V = \sqrt{2gh}$ …(iii)



This is required velocity of efflux

Applying continuity equation between 3 and 4 cross-section,

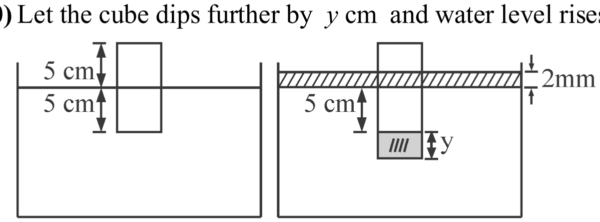
$$aV = a_1V_1$$

Again applying Bernoulli's equation between (iii) and (iv)

$P_{atm} + \frac{1}{2}(2\rho)V^2 + 2\rho g \times \frac{h}{2} = P_{atm} + \frac{1}{2}(2\rho)V_1^2 + 0 \Rightarrow V_1^2 = 3gh$

$$a_1 = \frac{aV}{V_1} = \frac{\sqrt{6} \cdot \sqrt{2gh}}{\sqrt{3gh}} = 2\,\text{cm}^2$$

39.(300) Let the cube dips further by y cm and water level rises by 2 mm.



Initial situation Final situation

Then equating the volume (/// volume = \\\ volume in figure)

$\Rightarrow$ Volume of water raised = volume of extra depth of wood

$\Rightarrow$ $100\,y = (1500-100)\dfrac{2}{10} = 1400\times\dfrac{2}{10} = 280$ $\therefore$ $y = 2.8\,\text{cm}$

$\therefore$ Extra upthrust

$\rho_{\text{water}} \times (2.8+0.2)\times 100\,g = \text{mg}$

$\Rightarrow$ $m = 300\,\text{gm}$

40.(0.45) Pressure of gas inside the balloon is same as the pressure of surrounding. Also gas inside the balloon obeys isothermal process, then :

$(P_0+\rho gh)V_1 = P_0V_2$ $\therefore$ $V_2 = \left(1+\dfrac{10^3\times 10\times 40}{10^5}\right)\times 0.09 = 0.45\ \text{m}^3$

41.(0.29) Force of buoyancy

$F_B = A[x\rho_0 + (L-x)\rho_0]g$

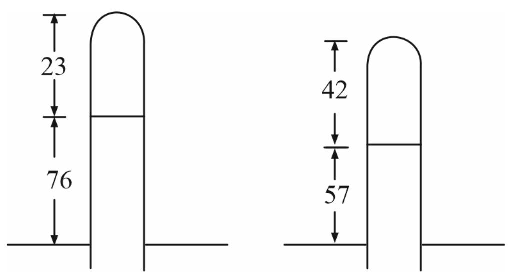
Weight $W = LA\rho_s\, g$

For equilibrium $F_B = W$

$\Rightarrow$ $LA\rho_s g = A[x\rho_0 + (L-x)\rho_0]g$ $\Rightarrow$ $L\rho_s = x\rho_0 + (L-x)\rho_w$

$\Rightarrow$ $\rho_s = \dfrac{x}{L}(\rho_0-\rho_w)+\rho_w$ $\Rightarrow$ $\dfrac{x}{L} = \dfrac{\rho_w-\rho_s}{\rho_w-\rho_0} = \dfrac{1000-800}{1000-300} = \dfrac{2}{7}$

42.(10.50) $76\times L = 42(76-57)$ $\Rightarrow L = 10.5$



$10.5\,\text{cm}\times 1\,\text{cm}^2 = 10.5$

43.(11.11) $1000\times\dfrac{4}{3}\pi r^3 g - 500\times\dfrac{4}{3}\pi r^3 g - 6\pi\eta r v = 500\times\dfrac{4}{3}\pi r^3 a$

$\dfrac{0.2}{3} = 6\times 1\times 10^{-4}\,v$ **;** $\dfrac{200}{18} = v$

44.(10) $R = 2\,\text{cm}$

$\Delta P = \dfrac{4S}{r} = \dfrac{4\times 0.05}{2\times 10^{-2}} = 10\ \text{Pa}$

45.(3) Let v be the velocity of the movable plate and F is equal to viscous force

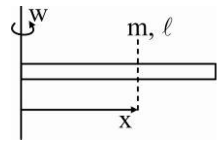
$F = \left[\eta_1\dfrac{v}{h_1} + \eta_2\dfrac{v}{h-h_1}\right]A$ $\Rightarrow$ $\dfrac{dF}{dh_1} = 0$ $\therefore$ $h_1 = \dfrac{h}{3}$

PROPERTIES OF MATTER

1.(A) Tension at point $x = mw^2 r$

$$T = \left\{\frac{m}{\ell}(\ell - x)\right\} \cdot w^2 \left\{\frac{\ell + x}{2}\right\}$$

$$T = \frac{mw^2}{2\ell}\left(\ell^2 - x^2\right)$$



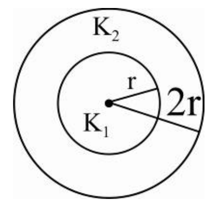
2.(B) $dq = msdT \quad \Rightarrow \quad q = \int_1^2 1 \times aT^3 dT = \frac{15a}{4}$

3.(B) $\frac{d\ell}{\ell} = 1\% \qquad \therefore \frac{dA}{A} = 2 \times 1\% = 2\%$

4.(C) $d\ell = dx(\alpha \Delta T) \quad \Rightarrow \quad d\ell = \left[(3x+2) \times 10^{-6} \times 20\right] dx$

Integrating $\Delta\ell = 1.208\, cm \qquad \therefore$ length $= 2.0120\, m$

5.(C) $R_1 = \frac{\ell}{K_1 \pi r^2}$ & $R_2 = \frac{\ell}{3K_2 \pi r^2}$



Both rods are in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \quad \frac{4\pi r^2 K_{eq}}{\ell} = \frac{\pi r^2 K_1}{\ell} + \frac{3\pi r^2 K_2}{\ell} \qquad \therefore K_{eq} = \frac{K_1 + 3K_2}{4}$$

6.(B) $\pi R^2 l = n\pi r^2 l \ldots\ldots\ldots\ldots..(i)$ and $n(2\pi rl) = 2(2\pi Rl) \ldots\ldots\ldots\ldots(ii)$ Solve for n $\therefore$ $n = 4$

7.(B) $\frac{-10}{4} = -K(55 - T_o) \ldots\ldots\ldots..(i)$ $\left\{\begin{matrix}\text{Newtons low of cooling} \\ \text{average method}\end{matrix}\right\}$

$\frac{-10}{8} = -K(35 - T_o) \ldots\ldots\ldots..(ii)$ Solving (i) & (ii) $T_o = 15°C$

8.(B) $\frac{2.4 \times 10^8 \times 3 \times 10^{-4}}{3} - 1200 \times 10 = 1200a \Rightarrow a = 10 m/s^2$

9.(D) $ms\frac{d\theta}{dt} = \sigma A \in \left(T^4 - T_0^4\right) \Rightarrow m_1 = 3m_2 \Rightarrow r_1 = 3^{1/3} r_2$

10.(A) Error $= \frac{1}{2}\alpha\theta .\, 86400 = 4.32\ sec$

11.(A) According to Newton'slaw of cooling $\frac{dT}{dt} \propto \Delta T$

$$\Rightarrow \quad \frac{\tan\theta_1}{\tan\theta_2} = \frac{35-20}{25-20} = \frac{15}{5} = \frac{3}{1} \quad \Rightarrow \quad \tan^2\theta_1 = 9\tan^2\theta_2$$

$$\because \quad \sec^2\theta_1 = 1 + \tan^2\theta_1 \quad \Rightarrow \quad \sec^2\theta_1 = 1 + 9\tan^2\theta_2$$

12.(ABC) In the case of a perfectly rigid body and incompressible liquid, volume strain is zero.

13.(BCD) $W = ms$ or, $m = \frac{W}{s} = \frac{4.5}{0.09} = 50\, g$

The thermal capacity and the water equivalent of a body have the same numerical value. Also, $Q = 4.5 \times 8 = 36\, cal$

Since, the temperature remains constant, during the process of melting, no heat is exchanged with the calorimeter and hence, $Q = 15 \times 80 = 1200\,cal$. Hence, the correct choices are (B), (C) and (D).

14.(AC) $(\text{Stress})_s = \dfrac{F}{2A}$, $(\text{Stress})_{Cu} = \dfrac{F}{A}$ Given that $\dfrac{Y_S}{Y_{Cu}} = \dfrac{2}{1}$

$$\frac{(\text{strain})_s}{(\text{strain})_{Cu}} = \frac{(\text{stress})_s / Y_s}{(\text{stress})_{cu} / Y_{cu}} = \frac{(F/2A)}{(F/A)} \times \frac{1}{2} = \frac{1}{4}$$

So, options (A), (B) and (C) are correct.

15.(BCD) Thermal force = $YA\alpha d\theta = Y\pi r^2 \alpha d\theta$

$r_1 = r$, $r_2 = r\sqrt{2}$, $r_3 = r\sqrt{3}$, $r_4 = 2r \Rightarrow$ $F_1 : F_2 : F_3 : F_3 = 1:2:3:4$

Thermal stress = $Y\alpha d\theta$

As Y and α are same for all the rods, hence stress developed in each rod will be the same. As strain $= \alpha d\theta,$ so strain will also be the same. E = Energy stored $= \dfrac{1}{2} Y(\text{strain})^2 \times A \times L$

$\therefore$ $E_1 : E_2 : E_3 : E_4 = 1:2:3:4$ So, option (B) and (C) are correct.

16.(AD) $H \propto \dfrac{A}{\ell}(T_1 - T_2)$

(A) Temp. diff. quadrupled, c/s area halved, H doubles (correct)

(B) Temp. diff. doubled, length quadrupled, H is halved (incorrect)

(C) c/s area halved, length doubled, H becomes 1/4th (incorrect)

(D) Temp. diff. doubled, Area doubled, length doubled, H is doubled (correct)

17.(AD) $\gamma_{Hg} > \gamma_{Fe} \Rightarrow$ density of Hg decreases more on heating. Hence cube will be submerged more.

If fraction submerged is f

$$\rho_{Hg} Vfg = \rho_{Fe} Vg \Rightarrow f = \frac{\rho_{Fe}}{\rho_{Hg}}$$

Change of fraction $= \dfrac{\dfrac{\rho'_{Fe}}{\rho'_{Hg}} - \dfrac{\rho_{Fe}}{\rho_{Hg}}}{\dfrac{\rho_{Fe}}{\rho_{Hg}}} = \dfrac{1 + \gamma_{Hg} \times 25}{1 + \gamma_{Fe} \times 25} - 1$

$$= \frac{1 + 180 \times 10^{-6} \times 25}{1 + 35 \times 10^{-6} \times 25} - 1 = \frac{(180 - 35) \times 10^{-6} \times 25}{1} = 3.6 \times 10^{-3} \quad = 0.36\,\% \simeq 0.4\%$$

18.(BD)

19.(AD) $\dfrac{dQ}{dt} = eA\sigma(T^4 - T_0^4) = \text{same}$; $\dfrac{d\theta}{dt} = \dfrac{eA\sigma}{ms}(T^4 - T_0^4) = \text{different}$

20.(ACD) Considering heat flow at junction A

Heat in = heat out

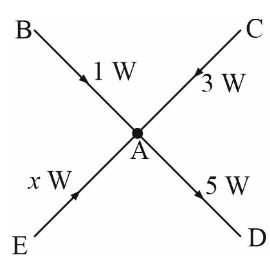
$x + 1 + 3 = 5$

$x = 1$

x is positive so heat flows in from E to A

$\therefore$ $T_E > T_A$

$\therefore$ $T_C > T_A > T_D$



$$T_B - T_A = T_E - T_A \therefore \qquad T_B = T_E$$

21.(ABCD) When a body is heated all dimension of the material as well as the enclosed lengths, areas and volumes also increase.

22.(BC) When temperature increases of a bimetallic strip, the one which is having more value of coefficient of expansion will expand more so that strip will bend toward the metal with low value of coefficient of expansion and it will bend toward high value of coefficient of expansion if it cooled.

23.(AD) By wein's displacement law we use

$$\lambda_m T = \text{constant} \quad \Rightarrow \quad \frac{\lambda_{mA}}{\lambda_{mB}} = \frac{T_B}{T_A} \quad \Rightarrow \quad \frac{T_A}{T_B} = \frac{\lambda_{mB}}{\lambda_{mA}} = \frac{800}{400} = 2$$

By Stefan's law, we use

$$\frac{E_A}{E_B} = \frac{A_A T_A^4}{A_B T_B^4} = \frac{4\pi r_A^2}{4\pi r_B^2}\left(\frac{T_A}{T_B}\right)^4 = 4$$

24.(D) $\dfrac{40-36}{10} = k\ (36-30); \dfrac{39-x}{10} = k\left(\dfrac{36+x}{2} - 30\right)$

$$\frac{4}{36-x} = \frac{8}{\left(\dfrac{x}{2} - 30 + 18\right)} \quad ; \quad \frac{x}{2} - 12 = 72 - 2x; \frac{5x}{2} = 72 + 12; \quad \frac{5x}{2} = 84; x = \frac{2 \times 84}{5} = \frac{168}{5} = 33.6$$

25.(D) $\dfrac{d\theta}{dt} = -\dfrac{kA}{ms}(T - T_0)$ Magnitude of slope will decrease with time.

26.(C) $\because \dfrac{40-36}{10} = k\ (38-30 \quad \Rightarrow \quad k = \dfrac{4}{10 \times 8} = \dfrac{1}{20}$

When the block is at 38 °C and room temperature is at 30°C the rate of heat loss

$$ms \times \frac{d\theta}{dt} = ms\ k\ (38-30)$$

Total heat loss in 10 minute $\Rightarrow \quad dQ = ms\ k\ (38-30) \times 10 \ = 2 \times \dfrac{1}{20} \times 8 \times 10 = 8 \text{ J}$

Now heat gained by the object in the said 10 minutes.

$$Q = ms\ \Delta\theta = 2 \times 4 = 8 \text{ J}$$

Total heat required = 8 + 8 = 16 J

27.(B) Material which is most ductile is easy to expand is used for making wire.

28.(C) Material which breaks just after proportional point.

29.(D) Material which retains permanent deformation.

30.(B) $V = V_0(1 + \gamma_L \Delta T) = (10)\ (100)\ [1 + 5 \times 10^{-5} \times 20] = 1000\ (1 + 0.001) = 1001 \text{ cm}^3 = 1001 \text{ cc}$

31.(B) Cross-sectional area of vessel at 40 °C

$$A = A_0(1 + 2\alpha_g \Delta T) = 100\ (1 + 2 \times 10^{-5} \times 20) = 100.04 \text{ cm}^2$$

$$\text{Actual height of liquid} = \frac{\text{Actual volume of liquid}}{\text{Cross-sectional area of vessel}}$$

$$= \frac{1001}{100.04} = (1001)\ (100 + 0.04)^{-1} = \left(\frac{1001}{100}\right)\left(1 + \frac{0.04}{100}\right)^{-1}$$

$$= \frac{(1001)}{100}\left(1 - \frac{0.04}{100}\right) = \frac{1}{100}(1001 - 0.4) = \frac{1000.6}{100} = 10.006 \text{ cm}$$

32.(B) $\because\ TV = SR(1 + \alpha_g \Delta T) = (TV)(1 - \alpha_g \Delta T)$

$\therefore\ SR = (TV)(1 + \alpha_g\ \Delta T)^{-1} = (10.006)(1 - 10^{-5} \times 20) = 10.006 - 0.002 = 10.004 \text{ cm}$

33. **[A – p r s ; B – p r s ; C – p ; D – p q r s]**

34. **[A-q; B-r ; C-s; D-p]**

(a) $\frac{10 \times A}{2}(100 - T) = \frac{20 \times A}{1}(T - 0)$; $100 - T - 4T = 0$; $T = 20$

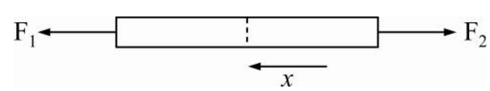
(b) $\frac{10 \times A(100)}{1} + \frac{20 \times A(100)}{1}$; $3000 \times 100 \times 10^{-4} = 30 \text{ W}$

(c) $R_{th} = \frac{(0.2 - 0.1)}{4\pi \times 0.2 \times 0.1} = 40$ SI units

`**(d)** $\frac{dt}{dx} = \frac{100 - 0}{10} = 10° \text{ C/m}$

35.(1) (T) Tension at any point $x = F_2 + \left(\frac{F_1 - F_2}{\ell}\right)x$ $\Rightarrow$ $d\ell = \frac{T}{AY}dx$

Integrating $\Delta\ell = \frac{(F_1 + F_2)\ell}{2AY}$ $\Rightarrow$ $\Delta\ell = 1 \times 10^{-9}\ m$ $\therefore$ $x = 1$



36.(2) Tension in ring = $\rho A\omega^2 r^2$ [A = C/s are of ring]

For breaking $\frac{\rho A\omega^2 r^2}{A} = \sigma$; $\omega^2 = \frac{\sigma}{r^2 \rho}$ $\therefore$ $\omega = 2\, rad/s$

37.(3) Let x *kg* of ice remaining

$(2)(15) \times 500 + (2 - x) \times 80{,}000 = (2.5)(25)(1000)$

$x \simeq 1.4\, kg$ $\therefore$ Water = 3 *kg*

38.(8) $\because$ Conduction rate $\propto \frac{A}{\ell}$

H $\propto \frac{radius^2}{length}$ $\therefore$ H (max) = $K\frac{4r^2}{\ell}$; $H_{min} = K\frac{r^2}{2\ell}$ $\therefore$ $\frac{H_{max}}{H_{min}} = 8$

39.(3) $dp = \frac{mg}{A}$

$K = \frac{-dp}{dv/v}$ $\Rightarrow$ $K\frac{3dr}{r} = -dp$ $\therefore$ $\frac{dr}{r} = -\frac{mg}{3AK}$ $n = 3$

40.(3) For no heat flow in AB temperature of Junction B must be $20°C$

$\because$ Material and cross-section are same

$\therefore$ Temperature gradient is also same

$\frac{80 - 20}{\ell_{BD}} = \frac{20 - 0}{\ell_{BC}}$ $\therefore$ $\frac{\ell_{BD}}{\ell_{BC}} = 3$

41.(4) $Q_{ab} = 200 \times 80 + 200 \times 1 \times 100 + (200 \times 1 \times 45)$

$Q_{loss} = m_s(540)$

$16000 + 20000 + 9000 = (540)\,m_s$; $\frac{45000}{540} = m_s$; $m_s = \frac{250}{3} = 83.33$

Steam left $= 87.33 - 83.33 = 4$

42.(9) Heat used for evaporation = 900 kJ

Mass evaporated = 0.2 kg $\quad \therefore\ L_v = \frac{900 \times 10^3}{0.2}$ J/kg

43.(4) Change in tension $= \Delta T \quad \therefore \quad 2\Delta T \sin 37° = \Delta mg$; $\Delta T = \frac{\Delta mg}{2\sin 37°}$; $\Delta l = \frac{\Delta TL}{AY}$

But $\quad \Delta l = L\alpha\Delta\theta \quad \therefore \quad \frac{\Delta mg}{2\sin 37°}\frac{L}{AY} = L\alpha\Delta\theta \quad \Rightarrow \quad \Delta m = \alpha\Delta\theta \times A \times Y \times 2\sin 37° / g$

$$= \frac{2 \times 10^{-5} \times 10 \times 10^{-6} \times 5 \times 10^{11} \times 2 \times 3}{5 \times 10} = 12$$

44.(3) From Newton's law of cooling

$$ms\left(\frac{\theta_1 - \theta_2}{t}\right) \propto \left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

In the first case $ms\left(\frac{50-45}{5}\right) \propto \left[\frac{50+45}{2} - \theta_0\right]$ …(i)

In second case $\left(\frac{45-41.5}{5}\right) \propto \left[\frac{45+41.5}{2} - \theta_0\right]$ …(ii)

From eq. (i) and (ii) $\theta_0 = 33.3°\text{C}$

45.(0.18)

KTG & THERMODYNAMICS

1.(B) Apply gas equation PV= nRT to identify decrease or increase in temp and volume for each process

2.(A) As the system is thermally insulated, $\Delta Q = 0$

Further as here the gas is expanding against vacuum (surroundings), the process is called free expansion and for it,

$\Delta W = \int PdV = 0$ [As for vacuum P = 0]

So in accordance with first law of thermodynamics, i.e. $\Delta Q = \Delta U + \Delta W$, we have

$0 = \Delta U + 0$, i.e. $\Delta U = 0$ or U = constant

3.(A) *ab* and *cd* are adiabatic

bc and *da* are isothermal. Hence $\left.\begin{aligned} P_bV_b &= P_cV_c \\ P_dV_d &= P_aV_a \\ P_cV_c^{\gamma} &= P_dV_d^{\gamma} \\ P_aV_a^{\gamma} &= P_bV_b^{\gamma} \end{aligned}\right\} \Rightarrow \frac{P_b}{P_c} = \frac{P_a}{P_d}$

4.(A) For adiabatic process (from A to B)

$\Delta W = -322J$

$\Delta U = -\Delta W = 322J$

For another process (from A to B)

$\Delta Q = \Delta U + \Delta W$

$100J = 322J + \Delta W \Rightarrow \Delta W = -222J$

5.(B) $C = C_v + \frac{RT}{V}\frac{dV}{dT}$; $R = \frac{3R}{2} + \frac{RT}{V}\frac{dV}{dT}$; $\frac{dV}{V} + \frac{1}{2}\frac{dT}{T} = 0 \Rightarrow VT^{1/2} =$ constant

6.(C) $\delta = \frac{\Delta V}{V\Delta T}$ from $PV = nRT \Rightarrow P\Delta V + V\Delta P = nR\Delta T$

Since '*P*' is constant $\Rightarrow P\Delta V = nR\Delta T \Rightarrow \delta = \left(\frac{nR}{P}\right)\cdot\frac{1}{T}$

7.(B) $Q = \Delta U + W \Rightarrow C = C_V + \frac{RT}{V}\frac{dV}{dT} \Rightarrow C = C_V - \frac{R}{\alpha V}(1 - \alpha V)$ **8.(D)** $U = 2\times\left(\frac{5R}{2}\right)T + 4\left(\frac{3R}{2}\right)T = 11RT$

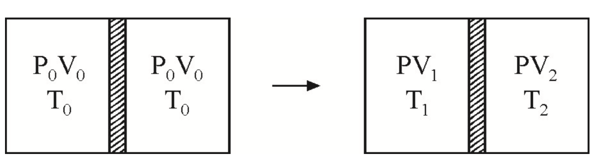
9.(B) $PV^{\gamma} =$ constant $\Rightarrow \frac{P}{\rho^{\gamma}} =$ constant $\Rightarrow \frac{P'}{p} = (32)^{7/5}$

10.(B) $U = a + bPV = a + nbRT \Rightarrow \frac{dU}{dT} = nbR$ But $\frac{dU}{dT} = \frac{nR}{\gamma - 1} \Rightarrow nbR = \frac{nR}{\gamma - 1} \Rightarrow \gamma = \frac{b+1}{b}$

11.(C) Along a to b, $\Delta u = 0$ and along b to c, $\Delta u = -w = -4j \Rightarrow a$ to c, $\Delta u = 0 + (-4) = -4J$ (for any path)

12.(A) $\sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{3RT}{M}}.(0.68) \Rightarrow \gamma = 1.3872,\ f = \frac{2}{\gamma - 1} \approx 5.16$

13.(C)



$2\frac{P_0V_0}{T_0} = \frac{PV_1}{T_1} + \frac{PV_2}{T_0}$ and $V_1 + V_2 = 2V_0$

$V_1 = x_1A$, $V_2 = x_2A \Rightarrow x_1 = 44$ *cm* and $x_2 = 40$ *cm*

14.(B) $\varepsilon_1 = 1-\frac{T_2}{T_1}$, $\varepsilon_2 = 1-\frac{T_3}{T_2}$, $\varepsilon_3 = 1-\frac{T_4}{T_3}$ as $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$ $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$; $T_2 = \left(T_1 T_3\right)^{\frac{1}{2}}$

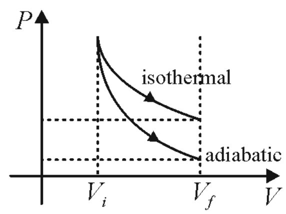
$$T_2 = \left(T_1 T_2^{\frac{1}{2}} T_4^{\frac{1}{2}}\right)^{\frac{1}{2}}; \quad T_2^{\frac{3}{4}} = \left(T_1 T_4^{\frac{1}{2}}\right)^{\frac{1}{2}} \Rightarrow T_2 = \left(T_1 T_4^{\frac{1}{2}}\right)^{\frac{1}{2}\times\frac{4}{3}} = T_1^{\frac{2}{3}} T_4^{\frac{1}{3}} \Rightarrow T_2 = \left(T_1^2 T_4\right)^{\frac{1}{3}}$$

Also $T_3 = \left(T_2 T_4\right)^{\frac{1}{2}} = \left(T_1^{\frac{2}{3}} T_4^{\frac{1}{3}} T_4\right)^{\frac{1}{2}} = \left(T_1^{\frac{2}{3}} T_4^{\frac{4}{3}}\right)^{\frac{1}{2}} = T_1^{\frac{1}{3}} T_4^{\frac{2}{3}} = \left(T_1 T_4^2\right)^{\frac{1}{3}}$

15.(A) $\beta = \frac{1-\eta}{n} \Rightarrow \beta = 9$ $\therefore$ $Q = 10\beta = 90J$

16.(ABC) For adiabatic process : $T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$

For isothermal process : $P_i V_i = P_f V_f$



17.(ABCD)

18.(ABD) In case of cyclic process : $\eta = 1-\frac{Q_R}{Q_g}$

Q_R = Heat rejected ; Q_q = Heat given

19.(CD) For A, D $T_1 V_a^{\gamma-1} = T_2 V_d^{\gamma-1} \Rightarrow \left(\frac{V_a}{V_d}\right)^{\gamma-1} = \frac{T_2}{T_1}$

For B, C $T_1 V_b^{\gamma-1} = T_2 V_c^{\gamma-1} \Rightarrow \left(\frac{V_b}{V_c}\right)^{\gamma-1} = \frac{T_2}{T_1} \Rightarrow \frac{V_b}{V_c} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$

So $\left(\frac{V_a}{V_d}\right) = \left(\frac{V_b}{V_c}\right)$, i.e., choices C and D are correct. Hence choice A and B are wrong.

20.(CD) $P = -mV + c \Rightarrow T = -\left(\frac{m}{nR}\right)V^2 + \left(\frac{c}{nR}\right)V \Rightarrow T_3 > T_1 = T_2$

$\therefore$ $1 \to 3 \to$ expansion process accompanied by heating

$3 \to 2 \to$ expansion process accompanied by cooling

21.(ABD) Degree of freedom of $He = 3$

Degree of freedom of $H_2 = 5$

Average degree of freedom $= \frac{3\times 2 + 5\times 2}{2+2} = 4$

Now $\gamma = 1 + \frac{2}{f} = \frac{3}{2}$; For adiabatic process $\Delta Q = 0$ and PV^{γ} = cost *or* $TV^{\gamma-1}$ = cost

From first law $\Delta U = \Delta w$ $[\because \Delta Q = 0]$

22.(AD) As, $W_{by} = Q = 0$, So U = constant so

$$2\times\frac{5}{2}R\times T_0 + 3\times 3R\times 2T_0 = \left(2\times\frac{5}{2}R + 3\times 3R\right)T$$

$$23RT_0 = 14RT \qquad \Rightarrow \qquad T = \frac{23T_0}{14}$$

By $\dfrac{n_1+n_2}{1-\gamma_{mix}} = \dfrac{n_1}{1-\gamma_1} + \dfrac{n_2}{1-\gamma_2}$ $\qquad \Rightarrow \lambda_{mix} = \dfrac{19}{14}$

23.(ABC) $\Delta Q = \Delta U + \Delta W$

For process A to B, $\Delta Q = \Delta W$

For process B to C, $\Delta Q = \Delta U$

For process C to D, $\Delta U = -\Delta W$

For process D to A, $\Delta U = -\Delta W$

24.(AC) $PV = 1\,RT$

$$(\alpha - \beta V^2)V = RT \quad ; \quad T = \frac{\alpha V}{R} - \frac{\beta V^2}{R} \quad ; \quad \frac{dT}{dV} = \frac{\alpha}{R} - \frac{3\beta V^2}{R}$$

For maximum value of T ; $\dfrac{dT}{dV} = 0 \qquad \Rightarrow \qquad V = \sqrt{\dfrac{\alpha}{3\beta}}$

25.(AD) Point A and C are on the same line passing through origin

$\Rightarrow \qquad \dfrac{P_A}{V_A} = \dfrac{P_C}{V_C} \qquad \ldots(i)$

Also $T_A = 200\ K = \dfrac{P_A V_A}{nR}$ and also $T_C = 1800\ K = \dfrac{P_C V_C}{nR}$

$\Rightarrow \qquad \dfrac{P_A V_A}{P_C V_C} = \dfrac{1}{9} \qquad \ldots(ii)$

From eq. (i) and (ii) $\dfrac{V_A}{V_C} = \dfrac{1}{3}$

26.(B) Heat gain by left part = heat lost by right part

$\Rightarrow \qquad \dfrac{3}{2}nR(T - T_0) = \dfrac{3}{2}nR(2T_0 - T) \Rightarrow T = \dfrac{3T_0}{2}$

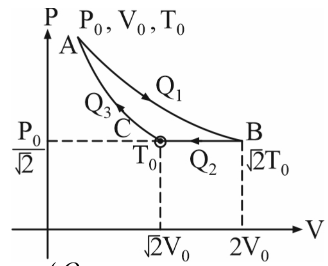
Let final pressure = p $\qquad \Rightarrow \qquad \dfrac{p}{T} = \dfrac{P_0}{T_0} \quad \Rightarrow \quad P = \dfrac{3P_0}{2}$

27.(C) Heat flow $= \dfrac{3}{2}nR\left(\dfrac{3T_0}{2} - T_0\right) = \dfrac{3}{4}nRT_0 = \dfrac{3}{4}P_0\dfrac{V_0}{2} = \dfrac{3}{8}P_0V_0$

28.(C) P (final in left) $= \dfrac{3P_0}{2} = P$ (final in Right part). So when pin is removed, piston will not move.

29.(C)

30.(C) $Q_1 = nC\Delta T - n\left(\frac{11R}{2}\right)(\sqrt{2}T_0 - T_0)$



$Q_2 = nC_p\Delta T = n\left(\frac{7R}{2}\right)(T_0 - \sqrt{2}T_0)$

$Q_3 = nRT \ln\left(\frac{V_f}{V_i}\right) = \frac{-nRT_0 \ln 2}{2}$

$\eta = \frac{\text{Total work done by the system}}{\text{Positive energy supplied to the system}}$

For cyclic process, total work done $(W) =$ total energy (Q

$$= \frac{\frac{nRT_0}{2}[4(\sqrt{2}-1) - \ln 2]}{\frac{1\ln RT_0}{2}(\sqrt{2}-1)}$$

$$= \frac{4(\sqrt{2}-1)\ \ln 2}{11(\sqrt{2}-1)} = \frac{4(0.40) - 0.7}{11(0.40)} = \frac{1.6 - 0.7}{4.4} = \frac{9}{44} \times 100\% = 20.5\%$$

31.(A)

32.(D) $\frac{p_1V_1}{T_1} = \frac{\left(p_1 + \frac{kx}{A}\right)(V_1 + Ax)}{T_2}$; $kx^2 + \left(p_1A + \frac{kV_1}{A}\right)x + \left(p_1V_1 - \frac{p_1V_2T_2}{T_1}\right) = 0$

$2000x^2 + 4600x - 480 = 0$; $x = 0.1\text{ m}$

$W_{\text{gas}} + W_{\text{atmosphere}} + W_{\text{spring}} = 0$; $W_{\text{gas}} - P_eAx - \frac{1}{2}kx^2 = 0$

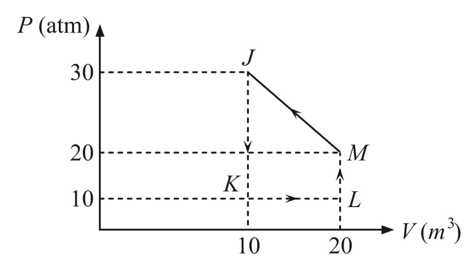
$W_{\text{gas}} = p_eAx + \frac{1}{2}kx^2 = 310\text{ J}$; $Q = \Delta U + p_eAx + \frac{1}{2}kx^2$; $\Delta U = \frac{f}{2}Nk\Delta T$

$\Delta U = \frac{f}{2}\frac{p_1V_1}{T_1} \cdot \Delta T = \frac{5}{2}\frac{10^5\,Pa \times 0.024\,\text{m}^3}{300\,\text{K}} \cdot 60\,\text{K} = 1200.0\text{ J}$

33. **[A – q, r, s ; B – r ; C – r ; D – s]**

34. **[A – q ; B – p s ; C – s ; D – q r]**

Temperature at :



$J = \frac{30 \times 10}{nR}$, $K = \frac{10 \times 10}{nR}$, $L = \frac{20 \times 10}{nR}$, $M = \frac{20 \times 20}{nR}$

For JK : $W = 0$; $\Delta U < 0 \Rightarrow \Delta Q < 0$

For KL : $W = 10 \times 10$; $\Delta U > 0 \Rightarrow \Delta Q > 0$

For LM : $W = 0$; $\Delta U > 0 \Rightarrow \Delta Q > 0$

For MJ : $W < 0$; $\Delta U < 0 \Rightarrow \Delta Q < 0$

35. **[A – r ; B – p ; C – s ; D – q]**

$W =$ Area of triangle $= \frac{1}{2} \times 2P_0 \times V_0 = P_0V_0$

For CA : $W = -P_0V_0 \Rightarrow \Delta U = \frac{3}{2}R\left[\frac{P_0V}{R} - \frac{2P_0V_0}{R}\right] = -\frac{3}{2}P_0V_0$ & $\Delta Q = -P_0V_0 - \frac{3}{2}P_0V_0 = \frac{-5}{2}P_0V_0$

For BC: $W = \frac{1}{2}\left[3P_0 + P_0\right]V_0 = 2P_0V_0 \Rightarrow \Delta U = \frac{3}{2}R\left[\frac{2P_0V}{R} - \frac{3P_0V_0}{R}\right] = -\frac{3}{2}P_0V_0$ & $\Delta Q = \frac{P_0V_0}{2}$

Maximum temperature will be for process BC.

For BC: $P = -\frac{2P_0}{V_0}V + 5P_0$ Using gas equation : $T = -\left(\frac{2P_0}{V_0R}\right)V^2 + \left(\frac{5P_0}{R}\right)V$

By using maxima/minima : $T_{max} = \frac{25}{8R}P_0V_0$

36. (2) For gas in A, $P_1 = \left(\frac{RT}{M}\right)\frac{m_A}{V_1}$ and $P_2 = \left(\frac{RT}{M}\right)\frac{m_A}{V_2}$ $\therefore$ $\Delta P = P_1 - P_2 = \left(\frac{RT}{M}\right)m_A\left(\frac{1}{V_1} - \frac{1}{V_2}\right)$

Putting $V_1 = V$ and $V_2 = 2V$ We get $\Delta P = \frac{RT}{M}\frac{m_A}{2V}$

Similarly for Gas in B, $1.5\Delta P = \left(\frac{RT}{M}\right)\frac{m_B}{2V}$ From eq. (I) and (II) we get $2m_B = 3m_A$

37.(7) For cylinder A. For cylinder B

$dQ = nC_P\ dT$ $dQ = nC_V\ dT'$

$nC_P\,dT = nC_V\ dT'$

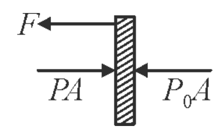
$\therefore$ $dT = \frac{C_P \times 30}{C_V} = 30 \times 1.4 = 42$ K

38.(1) Volume of the gas is constant V = constant $\therefore$ $P \propto T$

i.e. pressure will be doubled if temperature is doubled $\therefore$ $P = 2P_0$

Now let F be the tension in the wire. Then equilibrium of any one piston gives

$F = (P - P_0)A = (2P_0 - P_0)A = P_0A$



39.(6) $\frac{5}{2}nR\Delta T = \frac{1}{2}n \times Mu^2$ $\Rightarrow$ $\Delta T = \frac{Mu^2}{5R} = \frac{30\times10^{-3}\times10^4}{5\times R} = \frac{60}{R} = \frac{10}{R}\times6 \Rightarrow x = 6$

40.(100) Process is polytropic $C = C_v - \frac{R}{m-1}$

$$\frac{R}{2} = \frac{3}{2}R - \frac{R}{m-1} \Rightarrow m = 2$$

$$PV^2 = C$$

$$40\times V_0^2 = P(2V_0)^2$$

$$P = 10\ kPa$$

$$\frac{PV}{T} = \frac{P_0V_0}{T_0} \Rightarrow \frac{10\times2V_0}{T} = \frac{40\times V_0}{200} \Rightarrow T = 100\text{ K}$$

41.(2) $n_1C_{v_1}dT + n_2C_{v_2}dT + PdV = 0$

$\frac{1}{2}\times2RdT + 4\times\frac{7R}{4}dT + 4RT\frac{dV}{V} = 0$; $2\int\frac{dT}{T} + \int\frac{dV}{V} = 0$

$2\ln\left(\frac{T}{300}\right) + \ln\left(\frac{1}{4}\right) = 0$; $\ln\frac{T}{300} = \ln 2$

$T = 600$ K

$\Delta U = \frac{1}{2} \times 2R \times (600-300) + 4 \times \frac{7R}{4}(600-300)$ $\quad = 8R \times 300 = 8 \times \frac{25}{3} \times 300 = 2 \times 10^4$ J

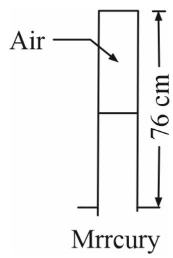
42.(22) Initial pressure of the gas $= P_0 = (P_{atm} - 40)$ cm of Hg = 36 cm Hg

Final pressure of the gas $= 1.5\, P_0 = 54$ cm ofHg (sicne process is isochoric)

Difference in height $= (76-54) = 22$ cm of Hg

43.(0.25) Pressure of air $(P) = \rho_{Hg} g\ (76-x)$

$\frac{P}{V} = \frac{\rho_{Hg} g}{A} =$ constant



Molar heat capacity (C)

$= \frac{R}{\gamma - 1} - \frac{R}{n-1}$

$= \frac{R}{1.4-1} - \frac{R}{-1-1} = 3R$

$\therefore\ \Delta Q = nC\Delta T = (10^{-3})\left(3 \times \frac{25}{3}\right)(10) = 0.25\,\text{J}$

44.(545) Let energy produced by fuel be x

Real engine

$\frac{20}{100} x = \frac{1}{2} m(4)^2$ . . .(i)

Carnot engine

$\eta x = \frac{1}{2} m(6)^2$ . . . (ii)

Equation $(ii) \div (i)$

$\frac{\eta}{0.2} = \left(\frac{6}{4}\right)^2 \Rightarrow \eta = 0.45$

Also

$\eta = 1 - \frac{T_C}{T_H} \Rightarrow \frac{T_C}{T_H} = 0.55$

$T_H = \frac{300}{0.55} \simeq 545k$

45.(3) **Carnot Engine**

$\eta = 1 - \frac{T_C}{T_H} \Rightarrow \frac{T_C}{T_H} = 1 - \eta$

Carnot refrigerator

$\propto = \frac{T_C}{T_H - T_C} \Rightarrow \propto = \frac{T_C / T_H}{1 - T_C / T_H}$

$\propto = \frac{1-\eta}{1-(1-\eta)} = \frac{1-\eta}{\eta} = \frac{0.75}{0.25}$

$= 3$

SIMPLE HARMONIC MOTION

1.(B) When collision occurs then velocity of both the body get interchanged and hence

$$T = \frac{T_{spring}}{2} + \frac{T_{spend}}{2} = \frac{1}{2}2\pi\sqrt{\frac{m}{k}} + \frac{1}{2}\pi\sqrt{\frac{l}{g}}$$

2.(C) Amplitude of damped oscillator is given by $A = A_0 e^{\frac{bt}{2m}}$

After, $5s, 0.9\,A_0 = A_0 e^{\frac{b(5)}{2m}}$ $\Rightarrow$ $0.9 = e^{-b\frac{(15)}{2m}}$ . . . .(i)

After, $10s\ A = A = A_0 e^{-b\frac{(15)}{2m}}$ $\Rightarrow$ $A = A_0 (e^{-\frac{5b}{2m}})^3$ . . . (ii)

From equation (i) and (ii), we get : $A = 0.729 A_0$ Hence, $\alpha = 0.729$

3.(B) $T = 2\pi\sqrt{\frac{L}{g}} = 2$ sec $\Rightarrow$ $T' = 2\pi\sqrt{\frac{L'}{g}} = 8$ sec $\Rightarrow$ $\eta = \frac{T'}{T} = 4$

4.(D) For upper half of oscillations, the block oscillates only with the upper spring and for the lower half of oscillation both springs are in parallel.

$\Rightarrow$ Period $= \frac{1}{2}2\pi\sqrt{\frac{m}{k}} + \frac{1}{2}2\pi\sqrt{\frac{m}{2k}}$ $\Rightarrow$ $T = \pi\sqrt{\frac{m}{k}} + \pi\sqrt{\frac{m}{2k}}$

5.(A) $x = 2a\sin(\omega t + \phi) \Rightarrow v = 2a\omega\cos(\omega t + \phi)$

At $t = 0,\ x = a, v = a\sqrt{3}$ $\Rightarrow$ $\sin\phi = \frac{1}{2}$ and $2\omega\cos\phi = \sqrt{3}a$ $\Rightarrow$ $\omega = 1\ rad/s$

$\phi = \pi/6$ $\Rightarrow$ $x = 2a\sin(t + \pi/6) = 2a\left[\sin t\cos\frac{\pi}{6} + \cot\sin\frac{\pi}{6}\right] = a\left(\sqrt{2}\sin t + \cos t\right)$

6.(B) Total energy $= U(0) + \frac{1}{2}KA^2 \Rightarrow 9 = 5 + \frac{1}{2}K(1)^2 \Rightarrow K = 8\,N/m$ Time period $= 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{2}{8}} = \pi\ sec$

7.(C) Use $x = A\sin\omega t$ find min. time when $x = \frac{A}{\sqrt{2}}$ and $x = A/2$ after that compare them

8.(D) For damped harmonic motion

$ma = -kx - mbv$ or $ma + mbv + kx = 0$

Solution of above equation is $x = A_0 e^{-\frac{b\tau}{e}} \sin\omega t$; with $\omega^2 = \frac{k}{m} - \frac{b^2}{4}$

Where, amplitude drops exponentially with time. i.e., $A_\tau = A_0 e^{-\frac{b\tau}{2}}$

Average time τ is that duration when, amplitude drops by 63% i.e., becomes A_0/e.

Thus, $A_\tau = \frac{A_0}{e} = A_0 e^{-\frac{b\tau}{2}}$ or $\frac{b\tau}{2} = 1$ or $\tau = \frac{2}{b}$

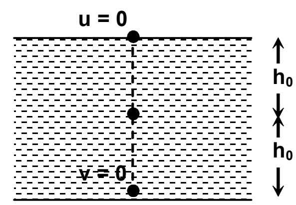
.

9.(CD) In case of S.H.M net force is always opposite to displacement from mean position.

10.(ABCD) $\hat{y} = -\hat{a}$ as Y increases V decrease and U increase

11.(AC) Net force on the ball will be zero at $\rho = \rho_0$ or, $\alpha h_0 = \rho_0$ or, $h_0 = \frac{\rho_0}{\alpha}$

i.e. the mean position is at a depth $h_0 = \frac{\rho_0}{\alpha}$



Net force at a depth $h_0 + x$ will be

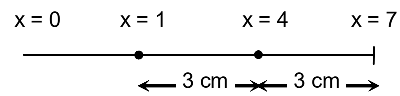
$F = (\rho - \rho_0)Vg\uparrow$ or, $F = \alpha x Vg\uparrow$

F is proportional to $-x$

Thus motion of the ball is simple harmonic $h_{max} = 2h_0 = \frac{2\rho_0}{\alpha}$

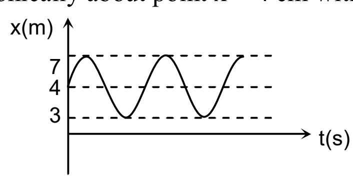
12.(ABD) $K = \frac{E}{4}$ $U = \frac{3E}{4}$ $\Rightarrow$ $x = \pm\frac{\sqrt{3}}{2}A$

13.(BD) The motion of the particle is somewhat like.



The minimum value of x can be 4 – 3 = 1 cm and maximum value of x can be 4 + 3 = 7 cm

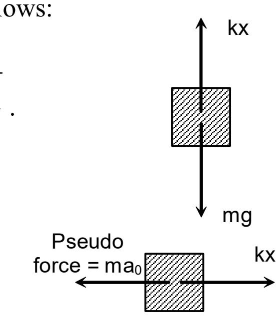
i.e., the particle oscillates simple harmonically about point x = 4 cm with amplitude 3 cm.



The x-t graph will be as show

14.(ABC) Free body diagram of the truck from non-inertial frame of reference will be as follows:

This is similar to a situation when a block is suspended from a vertical spring.



Therefore, the block will execute simple harmonically with time period $T = 2\pi\sqrt{\frac{m}{k}}$.

Amplitude will be given by $A = x = \frac{ma_0}{k}$ $(ma_0 = kx)$

Energy of oscillation will be $E = \frac{1}{2}kA^2 = \frac{1}{2}k\left(\frac{ma_0}{k}\right)^2 = \frac{m^2a_0^2}{2k}$

15.(BD) $x_1 = -A\cos\omega t$ and $x_2 = A\sin\omega t$

Equating $x_1 = x_2$, we get : $-A\cos\omega t = A\sin\omega t$

or, $\tan\omega t = -1$ or, $\omega t = \frac{3\pi}{4}$ or, $\left(\frac{2\pi}{T}\right)t = \frac{3\pi}{4}$ or, $t = \frac{3T}{8}$ $\Rightarrow$ $x_2 = A\sin\frac{3\pi}{4} = \frac{A}{\sqrt{2}}$

16.(A) E changes equilibrium position only.

17.(ABD) Description of motion is completely specified if we know the variation of x as a function of time. For simple harmonic motion, the general equation of motion is $x = A(\omega t + \delta)$. As ω is given, to describe the motion completely, we need the values of A and δ.

From option (b) and (d), we can have the values of A and δ directly.

For option (a), we can find A and δ if we know initial velocity and initial position. Option (c) cannot given the values A of and δ so it not the correct condition.

18.(ABCD) Period of oscillation changes as it depends on mas and becomes three times. The amplitude of oscillation does not change, because the new object is attached when the original object is at rest. Total energy does not change as at extreme position the energy is in the form of potential energy stored in spring which is independent of mass, and hence maximum; KE also does not change but as mass changes the maximum speed changes.

19.(ACD) The only external horizontal force acting on the system of the two blocks and the spring is F. Therefore, acceleration of the centre of mass of the system is equal to $F / m_1 + m^2$.

Hence, centre of mass of the system moves with a constant acceleration. Initially there is not tension in the spring, therefore at initial moment m_2 has an acceleration F / m_2 and it starts to move to the right. Due to its motion, the spring elongates and a tension is developed. Therefore, acceleration of m_2 decreases while that of m_1 increases from zero, initial value.

The blocks start to perform SHM about their centre of mass and the centre of mass moves with the acceleration calculated above. Hence option (b) is correct.

Since the blocks start to perform SHM about centre of mass, therefore the length of the spring varies periodically. Hence, option (a) is wrong.

Since magnitude of the force F remains constant, therefore amplitude of oscillations also remains constant. So option (c) also wrong.

Acceleration of m_2 is maximum at the instant when the spring is in its minimum possible length, which is equal to its natural length. Hence, at initial moments, acceleration of m_2 is maximum possible.

The spring is in its natural length, not only at initial moment but at time $t = T, 2T, 3T, ...$ also, where T is the period of oscillation. Hence, option (d) is wrong.

20.(AB) When point of suspension of pendulum is moved upwards, $g_{eff} = g + a,\ g_{eff} > g$ and as $T \propto 1/\sqrt{g_{eff}}$, hence T, decreases, i.e., choice (a) is correct.

When point of suspension of pendulum is moved downwards and $a > 2g$, then T decreases, i.e., choice (b) is also correct. In case of horizontal acceleration

$g_{eff} = \sqrt{g^2 + a^2}, i.e. g_{eff} > g$

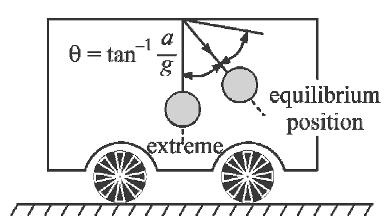
i.e., again, T decreases

21.(BCD) Due to the Pseudo force on block (considered external) its mean position will shift to a distance mg/K above natural length of spring as net force now is mg is upward direction so total distance of block from new mean position is 2mg/K which will be the amplitude of oscillations hence option (C) is correct. During oscillations spring will pass through the natural length hence option (D) is correct. As block is oscillating under spring force and other constant forces which do not affect the SHM frequency hence option (B) is correct.

22.(BC) Both will oscillate about equilibrium position with amplitude $\theta = \tan^{-1}\left(\frac{a}{g}\right)$ for any value of a.

If $a << g$, motion will be SHM, and then

Time period will be $2\pi\sqrt{\frac{l}{\sqrt{a^2 + g^2}}}\ 2$



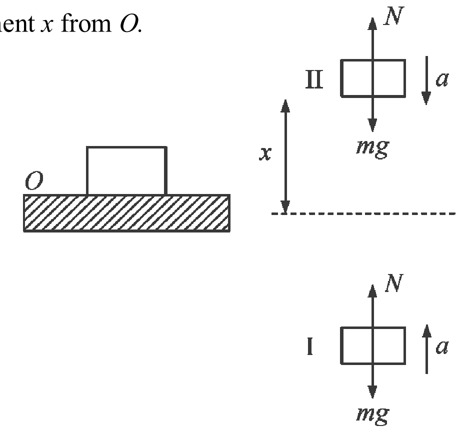
23.(AC) Let O be the mean position and a be the acceleration at a displacement x from O.

At position *I*, $N - mg = ma$ $\quad \therefore \quad N \neq 0$

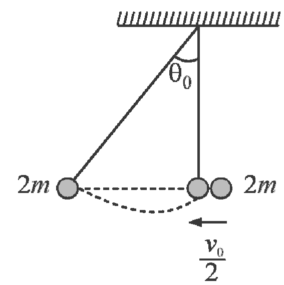
At position II, $mg - N = ma$

For $N = 0$ (loss of contact), $g = a = \omega^2 x$

Loss of contact will occur for amplitude $x_{max} = g/\omega^2$ at the highest point of the motion.



24.(AB) The time period of simple harmonic pendulum is independent of mass, so it would be same as that $T = 2\pi\sqrt{1/g}$. After collision, the combined mass acquires a velocity of $v_0/2$ as a result of this velocity, the mass (2m) moves up and at an angel θ_0 (say) with vertical, it stops, this is the extreme position of bob.



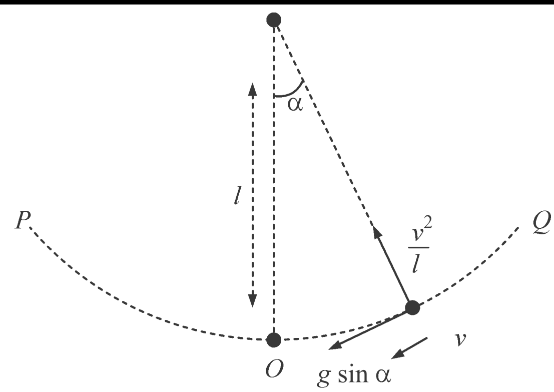
From work-energy theorem, $\Delta K = W_{total}$

$$0 - \frac{2m}{2}\left(\frac{v_0}{2}\right)^2 = 2mgl(1-\cos\theta_0) \quad ; \quad \frac{v_0^2}{8gl} = 1 - \cos\theta = 2\sin^2\frac{\theta_0}{2}$$

$$\sin\frac{\theta_0}{2} = \frac{v_0}{4\sqrt{gl}} \quad ; \quad \text{If } \theta_0 \text{ is small, } \sin\frac{\theta_0}{2} = \frac{\theta_0}{2} \Rightarrow \theta_0 = \frac{v_0}{2\sqrt{gl}}$$

So, the equation of simple harmonic motion is $\theta = \theta = \theta_0 \sin(\omega t)$

25.(AD) Statement (a) is correct. At any position O and P or between O and Q, there are two accelerations – a tangential acceleration $g\sin\alpha$ and a centripetal acceleration v^2/l (because the pendulum moves along the arc of a circle or radius l), where l is the length of the pendulum and v its speed at that position. When the bob is at the mean position O, the angle $\alpha = 0$, therefore $\sin\alpha = 0$; hence, the tangential acceleration is zero. But at O, speed v is maximum and the centripetal acceleration v^2/l is directed radially towards the point of support. When the bob is at the end points P and Q, the speed v is zero, hence the centripetal acceleration is zero at the end points, but the tangential acceleration is maximum and is directed along the tangent to the curve at P and Q. The tension in the string is not constant throughout the oscillation. At any position between O and end point P or Q, the tension in the string is given by $T = mg\cos\alpha$



At the end point P and Q, the value of α is the greatest, hence the tension is the least. At the mean position O, $\alpha = 0$ and $\alpha = 1$ g which is the greatest ; hence tension is greatest at the mean position.

26.(BD) Initially, $\frac{1}{2}kA^2 = \frac{1}{2}mv_0^2 \Rightarrow A = \sqrt{\frac{m}{k}}v_0$

Next, $mv_0 = 2mv \quad \Rightarrow \quad v = v_0/2 \quad \Rightarrow \quad \frac{1}{2}kA'^2 = \frac{1}{2}(2m)\left(\frac{v_0}{2}\right)^2$

$\Rightarrow \quad A' = \frac{A}{\sqrt{2}} \quad \Rightarrow \quad f' = \frac{1}{2\pi}\sqrt{\frac{k}{2m}} = \frac{f}{\sqrt{2}}$

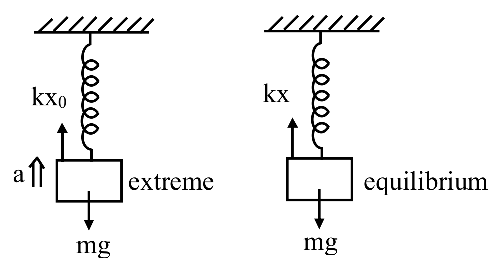
27.(AC) The situation is similar as if a block of mass m is suspended from a vertical spring and a constant force mg acts downwards. Therefore, in this case also block will execute SHM with time period $T = 2\pi\sqrt{\frac{m}{k}}$

At compression x ; $F = kx \Rightarrow x = \frac{F}{k}$

This is also the amplitude of oscillation.
Hence A = F/k
At mean position speed of the block will be maximum. Applying work energy theorem

$$F.x = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2Fx - kx^2}{m}} \quad ; \quad x = \frac{F}{k} \quad ; \quad V_{\max} = \frac{F}{\sqrt{mk}}$$

28.(C) 29.(B)



$$w = \sqrt{\frac{K}{m}} = \sqrt{\frac{800}{2}} = \sqrt{400} = 20\ rad/s$$

When car is accelerated let elongation is x_0

$$K x_0 - mg = ma$$

$\Rightarrow \quad 800\ x_0 - 20 = 20 \quad \Rightarrow \quad x_0 = 5\ cm$

When acceleration ceases let elongation of spring in equilibrium position $k\,x = mg \Rightarrow 800\,x = 20$

$\Rightarrow \quad x = 2.5\ cm \qquad$ Hence amplitude $= x_0 - x = 2.5\ cm$

Initially the block is at right extreme position. Hence initial phase $= \pi/2$.

30.(A) At $x = 0$, magnitude of displacement from means position $= 2m$ $\Rightarrow$ $2m/s^2 = \omega^2(2)$ $\Rightarrow$ $\omega = 1\ rad/s$

$\Rightarrow$ Time period $= \dfrac{2\pi}{\omega} = 2\pi\ sec$

31.(B) $a = (2 - x)m/s^2$ $\Rightarrow$ $v\dfrac{dv}{dx} = (2 - x)$

$$\Rightarrow \int_{v(0)}^{v(2)} v\,dv = \int_0^2 (2 - x)\,dx$$

$$\Rightarrow \frac{(v(2))^2 - (v(0))^2}{2} = 2(2) - \frac{(2)^2}{2} \Rightarrow \frac{1}{2}m\left[(v(2))^2 - (v(0))^2\right] = 4J$$

32.(D) As B is at its equilibrium position and moving towards negative extreme at $t = 0$

So $y - 4 = 2\sin(2\pi t + \pi)$ $\Rightarrow$ $y = 4 - 2\sin(2\pi t)$

33. **[A – q s ; B – p r; C – q s ; D – q s]**

When u is min equilibrium is stable and particle performs SHM. When u is max equilibrium is unstable.

34.(2) The effective acceleration of a bob in water = $g' = g\left(1 - \dfrac{d}{D}\right)$ where d & D are the density of water & the bob respectively. $\dfrac{D}{d}$ = specific gravity of the bob.

Since the period of oscillation of the bob in air & water are given as $T = 2\pi\sqrt{\dfrac{\ell}{g}}$ & $T' = 2\pi\sqrt{\dfrac{\ell}{g'}}$

$\therefore$ $T/T' = \sqrt{\dfrac{g'}{g}} = \sqrt{\dfrac{g(1 - d/D)}{g}} = \sqrt{1 - \dfrac{d}{D}} = \sqrt{1 - \dfrac{1}{s}}$

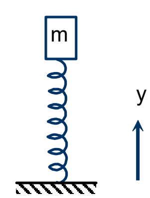
Putting $T/T' = 1/\sqrt{2}$, we obtain: $1/2 = 1 - 1/s$ $\Rightarrow$ $\dfrac{1}{s} = \dfrac{1}{2}$ $\Rightarrow$ $s = 2$

35.(6) $E = \dfrac{1}{2}m\omega^2 A^2$ $\Rightarrow$ $E = \dfrac{1}{2}m(2\pi f)^2 A^2$ $\Rightarrow$ $A = \dfrac{1}{2\pi f}\sqrt{\dfrac{2E}{m}}$

Putting E = K + U we obtain, $A = \dfrac{1}{2\pi(25/\pi)}\sqrt{\dfrac{2 \times (0.5 + 0.4)}{0.2}}$ $\Rightarrow$ A = 0.06 m.

36.(2) The loss of potential energy by the gravity = gain in potential energy in the spring

$mgx = \dfrac{1}{2}kx^2$ $\Rightarrow$ $x = \dfrac{2mg}{k}$

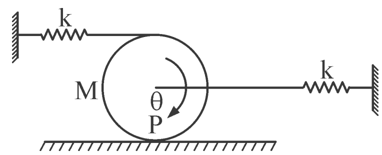


At equilibrium $\Sigma f_x = 0$; $mg - kx_0 = 0$ $\Rightarrow$ $x_0 = \dfrac{mg}{k}$

So, required ratio = 2 : 1

37.(3) For small angular displacement of cylinder.

The energy of system angular displacement θ is



$$E = \frac{1}{2}k\ (2R\theta)^2 + \frac{1}{2}k(R\theta)^2 + \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

Where v is the velocity of centre of mass and ω is the angular velocity of cylinder. Since E is constant.

$\therefore \quad \frac{dE}{dt} = 0 \quad \Rightarrow \quad 4kR^2\theta \cdot \frac{d\theta}{dt} + kR^2\theta\frac{d\theta}{dt} + Mv\frac{dv}{dt} + I\omega\frac{d\omega}{dt} = 0$

$\Rightarrow \quad 5kR^2\theta + MR^2\alpha + I\alpha = 0 \quad \Rightarrow \quad \alpha = -\frac{5kR^2}{(MR^2 + 1)}\theta$

where $I = \frac{MR^2}{2} \quad \Rightarrow \quad \alpha = \frac{5kR^2}{\frac{3}{2}MR^2}\theta$

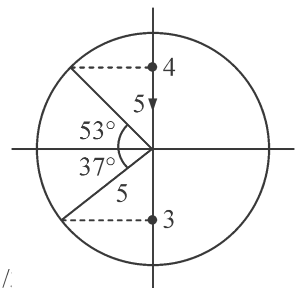
Compare it with $a = -\omega^2\theta$

Thus $\omega^2 = \frac{10k}{3M}$ or $T = 2\pi\sqrt{\frac{3M}{10K}}$

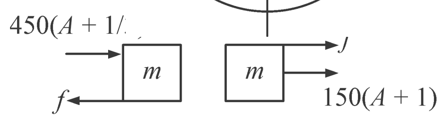
38.(5) By phasor diagram

Angle traced = 53° + 37° = 90°

$\therefore$ Time taken $= \frac{90°}{360°} \times T = \frac{T}{4} = 5s$



39.(4) 600 A + 300 = 2ma
$450A + 150 - f = ma$
$f = \mu mg = 150A$



40.(1) When block is displaced down by y, then level of liquid in beaker also rises.

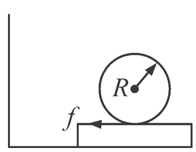
$\frac{A}{4}y = \left(A - \frac{A}{4}\right)y'$

So, upthrust $= \left(y - \frac{y}{3}\right)\frac{A}{4}3dg$

$\left(\frac{A}{4}\right)dha = -\left(y - \frac{4y}{3}\right)\frac{A}{4}(3d)g$

$a = -\left(\frac{4g}{h}\right)y \quad ; \quad T = 2\pi\sqrt{\frac{h}{4g}} = \pi\sqrt{\frac{h}{g}}$

41.(9) $\alpha = \frac{2f}{mR}$



$a_{CM} = \frac{f}{m}$

$a_{POC} = \frac{3f}{m} = A(10)^2 \cos(10)t$

For no slipping
$100A = 3\mu g = 9 \quad \Rightarrow \quad A = 9cm$

42 . $\left[T = 2\pi\sqrt{\frac{m(k_1 + 4k_2)}{k_1k_2}}\right]$

43. $\left[2\pi\sqrt{\frac{(\pi - 2)r}{g}}\right]$

44.(500) $A = A_0 e^{-bt/2m}$

$5 = 15e^{-\frac{bt}{2m}}$

$\frac{bt}{2m} = \ln 3$

$\frac{b}{2m} = \frac{\ell n 3}{1100} J \ \ \frac{m}{b} = \left(\frac{1100}{\ln 3}\right)\frac{1}{2} = \left(\frac{1100}{1.1}\right)\frac{1}{2}$

$= 500$

45.(5) $A = \frac{F_{ext}/m}{\omega^2 - \omega_0^2}$

$\omega^2 = \frac{F_{ext}}{m} + \omega_0^2$

$= \frac{F_{ext} + k}{m}$

$= \frac{1.7 + 6.3}{0.32}$

$= \frac{8}{0.32} = 25$

$\omega = 5$

WAVE MOTION

1.(B) $\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow T_1 = 507\,N$; $T_2 = 507 - 10000V$

$$\frac{240}{260} = \sqrt{\frac{507-10000V}{507}} \Rightarrow V = 0.0075\,m^3$$

2.(B) **3.(B)** $I \propto A^2$

4.(D) Compare with the standard equation of the wave. **5.(D)** Apply equation for Doppler's effect.

6.(B) Beats are produced due to the difference in apparent frequency of the two tuning forks.

7.(A) At 4 sec $V = 40\,m/s \Rightarrow f' = \frac{320}{320+40} \times 1000 = 889\,Hz$

8.(D) $\frac{f_1}{f_2} = \frac{340-17}{340-34} = \frac{19}{18}$

9.(C) $f_{max} = f_0\left(\frac{C + A\omega}{C}\right)$ and $f_{min} = f_0\left(\frac{C - A\omega}{C}\right)$, $\omega = \sqrt{\frac{k}{m}}$

$$= 990\left(\frac{330+10}{330}\right) = 1020\ Hz\ ;\ = \frac{990(330-10)}{330} = \sqrt{\frac{\omega}{4}} = 960\ Hz$$

10.(D) Along perpendicular bisector, path difference is zero, but phase difference between L_1 and L_2 is π , So, waves interfere destructively.

11.(AC) Only these two satisfies, $\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$

12.(ABCD)

$I_{eq} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

For maximum & minimum

$\cos\phi = \pm 1$

$\Rightarrow I_{min} = 0$ and $I_{max} = 4\,I$

13.(BC) $\frac{2\pi x}{3} = \frac{2\pi x}{\lambda} \Rightarrow \lambda = 3$ m

$120\,\pi t = 2n\pi t \Rightarrow n = 60$ Hz.

$v = ns = 180$ m/s $\Rightarrow v = \sqrt{\frac{I\ell}{m}} \Rightarrow T = 648$ N

Amplitude at a distance $x = 2a\sin\left(\frac{2\pi}{\lambda}\right)x = 4.2$ cm

14.(AC) $|V_p| = \left|\frac{dy}{dt}\right| = \left|\frac{2}{\left[(x-3t)^2+1\right]^2} \times 2(x-3t)(-3)\right|$ speed of particle at $t = 1$ and $x = 3$

$$= \left|\frac{2}{\left[0^2+1\right]^2} \times 2(3-3)(-3)\right| = 0$$

Speed of wave $= \frac{\text{Coeff of } t}{\text{coeff of } x} = \frac{3}{1} = 3\,cm/s$

15.(ABC) App. frequency of B as heard by A $= 500\left[\frac{300-0}{350-(-50)}\right] = 437.5\,Hz$

App. frequency of B as heard by O $= 500\left[\dfrac{300-(-25)}{350-(-50)}\right] = 468.75\ Hz$

App. frequency of A as heard by O is $500\left[\dfrac{350-25}{350}\right] = 464.28\ Hz$

Diff = $468.75 - 464.28 = 4.47$

App. frequency of A as heard by B $= 500\left[\dfrac{300-50}{350}\right] = 428.57\ Hz$

16.(AC) $\pi = \dfrac{2\pi}{\lambda}$ path diff (p) $\Rightarrow$ $p = \dfrac{\lambda}{2}$ and $\dfrac{2\pi}{\lambda} = \pi \Rightarrow p = 1m$

$f = \dfrac{\omega}{2\pi} = \dfrac{4\pi}{2\pi} = 2Hz$, at $x = 20\ cm$, $t = 4$ sec $\Rightarrow$ $y = 0.15 sin\left(4\pi(4) - \dfrac{\pi}{5}\right) = -0.15\ sin\dfrac{\pi}{3}$ **17.(BCD)**

18.(ABD) It is a known fact as well as experimentally and analytically verified that wave speed depends on the properties of the medium and is same for the entire wave. The particle velocity is given by

$$v_P = \frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$$

Where symbols have their usual meanings. It is clear form above expression that v_P depends upon amplitude and frequency of wave which are wave properties and are having different values for different particles at a particular instant.

19.(CD) Since the first wave and the third wave moving in the same direction have the phase angles ϕ and $(\phi + \pi)$, they superpose with opposite phase at every point of the vibrating medium and thus cancel out each other, in displacement, velocity, and acceleration. They in effect, destroy each other out. Hence we are left with only the second wave which progresses as a simple harmonic wave of amplitude A. the velocity of this wave is the same as if it were moving alone.

20.(BC) At any point on line *AB*, the phase difference between two waves is zero and hence waves will interference constructively.

Along *CD*, the phase difference changes and waves interference constructively and destructively and, hence sound will be loud, faint and so on.

21.(CD) $y_A = \sin[\omega t - k(AC)]$; $y_B = \sin[\omega t - \dfrac{\pi}{2} - k(BC)$

For maximum intensity at *C*

$k(BC - AC) + \dfrac{\pi}{2} = 2n\pi k$; $BC - AC = \left(n\lambda - \dfrac{\lambda}{4}\right) = 15, 35, 55, 75.....$

22.(AD) In both case (A) and (D) the source and observer are relatively at rest, thus neither of them is approaching or separating from each other. Effectively, it is the medium that moves in each of these cases. The received (apparent) frequency differs from the emitted frequency if and only if the time required for the wave to travel from the source to observer is different for different wavefronts. With a uniform steady motion of the medium, past the observer and source, the transit time from source to observer is the same for all wavefronts. Hence is follows that apparent frequency is equal to the true emitted frequency. Thus there is no Doppler effect. In case (B) and (C), Doppler effect will be observed as the source and observer have a relative speed and so they will approach or recede from each other.

23.(ACD) If intensity at points is *I*, then energy density at that point is *E* = *I*/*v*, where v is wave propagation velocity.

It means that $E \propto I$, Hence, the graph between *E* and *I* will be a straight line passing through the origin. Therefore (a) is correct and (b) is wrong. Intensity is given by :

$$I = 2\pi^2 n^2 a^2 pv$$

Hence,

$$E = 2\pi^2 n^2 a^2 \rho$$

It means that $E \propto n^2$

Hence, the graph between E and n will be parabola passing through origin, having increasing slope and symmetric about E-axis. Hence, option (d) is correct.

Particle maximum velocity is

$$u_0 = a\omega = 2\pi na \quad \Rightarrow \quad \pi na = \frac{u_0}{2} \quad ; \quad \text{Hence, } E = \frac{1}{2}\rho u_0^2$$

It means that graphs between E and u_0 will be a parabola, have increasing slope and will be symmetric about E-axis. Hence. Option (C) is also correct.

24.(C) 25.(A) 26.(A)

Mass per unit length of the string is

$m = Ad = (0.80\ mm^2) \times (12.5\ g/cm^3)$

$= (0.80 \times 10^{-6}\ m^2) \times (12.5 \times 10^3\ Kg/m^3) = 0.01 Kg/m$

Speed of transverse waves produced in the string

$$v = \sqrt{\frac{T}{M}} = \sqrt{\frac{64}{0.01\ Kg/m}} = 80\, m/s$$

The amplitude of the source is a = 1.0 cm and the frequency is $n = 20\,Hz$. The angular frequency is $\omega = 2\pi n = 40\pi s^{-1}$.

Also at t = 0, the displacement is equal to its amplitude, i.e., at t = 0, y = a. The equation of motion of the source is therefore.

$y = (1.0cm)\cos[(40\pi s^{-1})t]$ . . . .(i)

The equation of the wave travelling on the string along the positive X-axis is obtained by replacing t by $(t - x/v)]$ in equation (i). It is, therefore,

$y = (1.0\,cm)\cos[(40\pi s^{-1})\ \{t - (x/v)\}]$

$= (1.0\,cm)\cos[(40\pi s^{-1})t - \{(\pi/2)m^{-1}\}x]$ . . . .(ii)

The displacement of the particle at x = 50 cm at time t = 0.05 s is obtained from equation (ii)

$y = (1.0cm)\ \cos[(40\ \pi s^{-1})(0.05s)$

$-\{(\pi/2)m^{-1}\}(0.5\ m)]$

$= (1.0\ cm)\ \cos[2\pi - (\pi/4)]$

$= 1.0cm/\sqrt{2} = 0.71\ cm$

27. **[A – q s ; B – p r ; C – p r; D – q s]**

Situation (i) direction $\rightarrow$ same ; frequency $\rightarrow$ different

Situation (ii) direction $\rightarrow$ opposite ; frequency $\rightarrow$ same

28. **[A – q ; B – p ; C – r ; D – r]**

(A) $f = \dfrac{f_1 + f_2}{2}$ **(B)** $A' = 2A$

(C) Beat frequency $= |f_2 - f_1|$ **(D)** Ratio $= \dfrac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$

29.(1) The general equation of a wave travelling along the positive x-direction is y = f(x – ct) where c is the wave velocity.

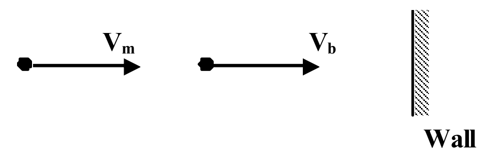
At t = 0, $y = \dfrac{1}{\sqrt{1+x^2}}$ (given) and the equation of the wave reduces to y = f(x)

Now, at t = 1s : $y = \dfrac{1}{\sqrt{2-2x+x^2}} = \dfrac{1}{\sqrt{1+(x-1)^2}} = f(x-1)$

Comparing the equation with y = f(x - ct) at t = 1s, we get c = 1 m/s.

30.(4) Required beat frequency = $|f_1 - f_2|$

Where, f_1 = apparent frequency for the motorist corresponding to the signals directly coming to him from source, and f_2 = apparent frequency for the motorist corresponding to the signals coming to him after reflection.



Now, $f_1 = f\left[\dfrac{V+V_m}{V+V_b}\right]$

Where f ′ is the frequency at which signals from sources are incident on wall.

$f' = f\left[\dfrac{V}{V-V_b}\right] \quad \Rightarrow \quad f_2 = f\left[\dfrac{V}{V-V_b}\right]\left[\dfrac{V+V_m}{V}\right] = f\left[\dfrac{V+V_m}{V-V_b}\right]$

Hence, the beat frequency = $|f_1 - f_2| = \dfrac{2V_b(V+V_m)f}{(V^2 - V_b^2)}$.

31.(2) The new position of the 8 kg block

T sin θ = 6400 N; T cos θ = 4800 N

Squaring and adding T = $\sqrt{(64)^2 + (48)^2}$ = 80 N

Now velocity of transverse wave = $\sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{80}{20\times10^{-4}}}$ = 2 × 10^{-2} m/s= 2 cm/s.

32.(3) mid point will be having maximum displacement (Antinode) when string will vibrate in 3^{rd} harmonic (in second harmonic mid point will be position of node)

33.(4) $y_R = y_1 + y_2 = 3A\cos(\omega t - kx) + A\cos(3\omega t - 3kx)$ ⇒ $y_R = 4A\sin^3(\omega t - kx)$ ⇒ $A_{Resultant} = 4A$

34.(3) $Y_{max} = Y_{1(max)} + Y_{2(max)}$

35.(2) Velocity of the wave,

$V = \sqrt{\left(\dfrac{T}{\mu}\right)} = \sqrt{\dfrac{(16\times10^5)}{0.4}} = 2000\ cm/s$

Time taken to reach to the other end $= \frac{20}{200} = 0.01\ s$

Time taken to see the pulse again in the original position $= 0.01 \times 2 = 0.02\ s$

36.(4) $\mu = 19.2 \times 10^{-3}\ kg/m$

From the free body diagram

$T - 4g - 4a = 0$; $T = 4(a+g) = (2+10) = 48\ N$ T

Wave speed :

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{48}{19.2 \times 10^{-3}}} = 50\ m/s$; So $n = 4$

37.(2) Given that

$x = 40\cos(50\ \pi t - 0.02\ \pi y)$ $\therefore$ particle velocity

$$v_p = \frac{dx}{dt} = (40 \times 50)\ \{-\sin(50\pi t - 0.02\ \pi y)\}$$

Putting $x = 25$ and $t = \frac{1}{200} s$

$v_p = -(2000\pi\ cm/s)\ \sin\left[50\pi\left(\frac{1}{200}\right) - 0.02\ \pi(25)\right] = 10\pi\sqrt{2}\ m/s$

38.(2) $a_{max} = \omega^2 A = g$; $\omega = \frac{2\pi v}{\lambda},\ v = \sqrt{\frac{F}{\mu}}$

$A_{min} = \frac{g\lambda^2\mu}{4\pi^2 F} = \frac{\lambda^2\mu}{4F} = 2 \times 10^{-3}\ m = 2\ mm$

39.(3) $f \propto \sqrt{T}$ for strings.

On increasing the tension by 1%

$f' = \sqrt{101T}$

$\frac{f'}{f} = \frac{\sqrt{1.01T}}{\sqrt{T}} = (1+0.01)^{\frac{1}{2}} = 1 + \frac{1}{200}$; Beat frequency, $f' - f = f\left(\frac{f'}{f} - 1\right) = 1$

Number of beats in $3s = 1 \times 30 = 30$

40.(7) $f_0 - f_c = 2$

$V\left[\frac{1}{2L} - \frac{1}{4L}\right] = 2\ or\ V/L = 8$ In the second case,

$f_0' - f_c' = \frac{V}{L} - \frac{V}{8L} = \frac{7V}{8L} = \frac{7}{8}(8) = 7$

41.(1) Intensity is given by $I = \frac{p_0^2}{2\rho v}$

Here v and ρ are same for both. And also given that I is same both. So pressure amplitude is also same for both.

42.(7) Intensity from a point source varies with distance as $I \propto \frac{1}{r^2}$

Let at distance $r_1 = 10m$, intensity is I_1

Then given $20 = 10\log\frac{I_1}{I_0}$ (i)

Let for $r = r_2$, sound level be zero. Then intensity at that point should be $I_2 = I_0$.

And $\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2 \Rightarrow \frac{I_1}{I_0} = \left(\frac{r_2}{r_1}\right)^2$ (ii)

From, Eqs. (i) and (ii), we get

$$20 = 10\log\left(\frac{r_2}{r_1}\right)^2 \Rightarrow 20 = 20\log\left(\frac{r_2}{r_1}\right) \Rightarrow \frac{r_2}{r_1} = 10 \Rightarrow 10r_1 = 7m$$

43.(8) The observer will hear a sound of the source moving away from him and another sound after reflection from the wall. The apparent frequencies of these sounds are

$$f_1 = \left(\frac{v}{v+u}\right)f,\ f_2 = \left(\frac{v}{v-u}\right)f$$

Number of beats $f_2 - f_1 = \left(\frac{v}{v-u} - \frac{v}{v+u}\right)f = \frac{2uvf}{v^2 - u^2} = \frac{2uf}{v} = 8$

44.(4) Here $I_1 = 1.0 \times 10^{-8}\ W/m^2$

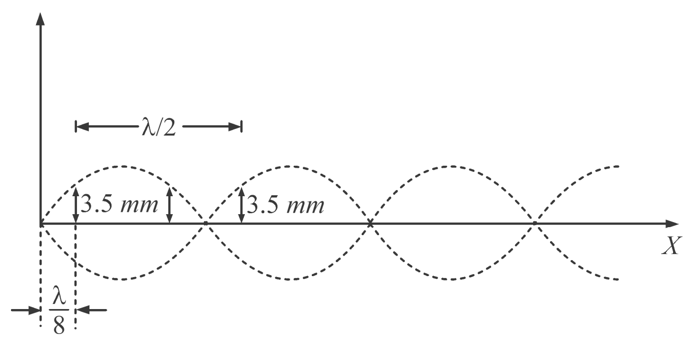
$r_1 = 5.0\ m,\ I_2 = ?, r_2 = 25m$

We know that $I \propto \left(\frac{1}{r^2}\right)$

$$I_1 r_1^2 = I_2 r_2^2 \quad ; \quad I_2 = \frac{I_1 r_1^2}{r_2^2} = \frac{1.0 \times 10^{-8} \times 25}{625} = 4.0 \times 10^{-10}\ W/m^2$$

45.(5) We can see from figure, in a stationary wave two successive of equal amplitude, if separated by equal distances then this distance must be $\frac{\lambda}{4}$. Thus we have

$$\frac{\lambda}{4} = 15cm \qquad \text{Or} \qquad \lambda = 60\ cm$$



Thus there are four loops in 120 *cm* length of string. This corresponds to 3rd overtone oscillations. As shown in figure if we consider origin *O* is at a node then the amplitude of a general medium particle, at a distance *x* from *O* can be given as

$R = A_0 \sin kx$

Where A_0 is the maximum displacement amplitude. First point from the origin where amplitude is 3.5 *mm* is at distance $\frac{\lambda}{8} = 7.5\ cm$. Thus we have

$$3.5 = A_0 \sin\left(\frac{2\lambda}{60} \times 7.5\right) \quad \text{or} \quad A_0 = \frac{3.5}{\sin(\pi/4)} = 3.5\sqrt{2}\ mm \approx 5mm$$

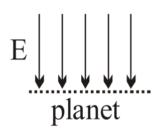
ELECTROSTATICS

1.(C) If another shell is kept upside down over it complete a sphere, net field should become zero.

2.(B) Net downwards acceleration on body of mass m = $\left(g+\frac{qE}{m}\right)$ = a_{net}

If E = uniform electric field in downwards direction

If it hits after time $t = \frac{2v}{a_{net}} = \frac{2v}{\left[g+\frac{qE}{m}\right]}$



at maximum height $v_f = 0$

$v^2_f = v^2_i - 2a_{net}h$ ⇒ In uniform field (Gravitation + Electric field time to reach highest point = t / 2]

$v^2 = 2\left[g+\frac{qE}{m}\right]h$

ΔV between ground and highest point : ΔV = (E) h

$a_{net} = \frac{2v}{t} \Rightarrow \left(g+\frac{qE}{m}\right) = \frac{2v}{t} \Rightarrow \frac{q}{m}E = \left(\frac{2v}{t}-g\right)$

$E = \frac{m}{q}\left(\frac{2v}{t}-g\right)$ and h = (average velocity) × t ⇒ $h = \frac{v}{2}\times t$

So $\Delta V = \frac{m}{q}\left(\frac{2v}{t}-g\right)\left(\frac{v}{2}t\right) = \frac{mv}{q}\left(v-\frac{gt}{2}\right)$

3.(C) If q denotes the charge in the medium from R to r, then $E = \frac{K(Q+q)}{r^2}$

q can be calculated by integrating the charge dq in a thin shell of radius r, thickness dr. This gives $q = 2\pi\alpha(r^2 - R^2)$

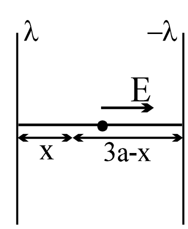
i.e, $E = \frac{K(Q+2\pi\alpha(r^2-R^2))}{r^2}$

Electric field will be uniform if coefficient of r is made zero ⇒ $\frac{kQ}{r^2} = 2\pi\alpha k\frac{R^2}{r^2}$ ⇒ $Q = 2\pi\alpha R^2$

4.(D) $E = \frac{\lambda}{2\pi\epsilon_0 x} + \frac{\lambda}{2\pi\epsilon_0(3a-x)} = -\frac{dV}{dx}$ ∴ $\int -dV = \frac{\lambda}{2\pi\epsilon_0}\int\frac{dx}{x} + \int\frac{dx}{3a-x}$

$= \frac{\lambda}{2\pi\epsilon_0}\left\{[\ln x]_a^{2a} - [\ln(3a-x)]_a^{2a}\right\} = \frac{\lambda}{2\pi\epsilon_0}[\ln 2 + \ln 2] = \frac{\lambda}{2\pi\epsilon_0}\cdot 2\ln 2 = V_A - V_B$

∴ work = $(V_A - V_B)\,q_0 = \frac{\lambda q_0 \ln 2}{\pi\epsilon_0}$



5.(B) For earthed conductor [the inner shell], V = 0

Here $V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{3r}+\frac{q'}{r}\right]$ where q′ is charge that would appear on inner shell as it is grounded ⇒ q′ = –q/3

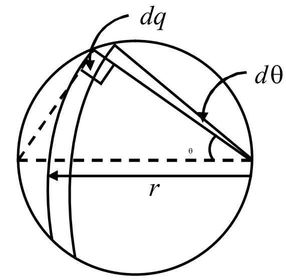
Hence, the charge flown to earth = 0 – (–q/3) = q/3

6.(A) $r = 2R\cos\theta$

$dV = k\dfrac{dq}{r}$

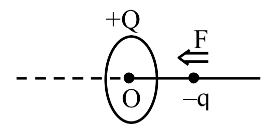
$dq = \sigma((2\theta)r)dr \quad \Rightarrow \quad dV = k\dfrac{\sigma(2\theta)2R\cos\theta(-2R\sin\theta\, d\theta)}{2R\cos\theta}$

$$V = \int dV = -4k\sigma R\int_0^{\pi/2}\theta\sin\theta\, d\theta = \frac{\sigma R}{\pi\in_0}.$$



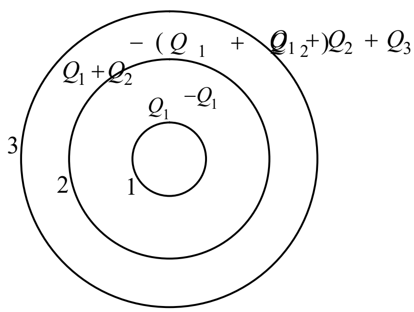
7.(B) The system can be treated as a system of two dipoles having dipole moment p = qa each. If you think a little, you will realize that the dipoles are perpendicular to each other. Obviously, the net dipole moment is $qa\sqrt{2}$.

8.(D) $E(x) = \dfrac{Qx}{4\pi\in_0\left(R^2+x^2\right)^{3/2}} \simeq \dfrac{Qx}{4\pi\in_0 R^3}$ (for $x << R$)



$F = \dfrac{Qq}{4\pi\in_0 R^3}x$ (Towards mean position) $\Rightarrow$ $T = 2\pi\sqrt{\dfrac{4\pi\in_0 mR^3}{Qq}}$

9.(B) Charge distribution is as shown :



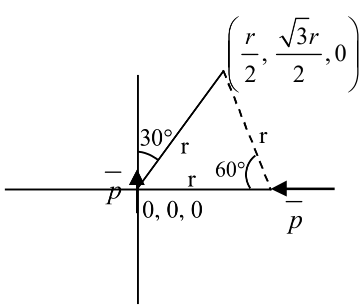
$\sigma_1 = \sigma_2 = \sigma_3 \Rightarrow \dfrac{Q_1}{4\pi R^2} = \dfrac{Q_1+Q_2}{4\pi(2R)^2} = \dfrac{Q_1+Q_2+Q_3}{4\pi(3R)^2}$

On solving we get ; $Q_1 : Q_2 : Q_3 = 1:3:5$

10.(C)

11.(A) Field at P due to outside changes $= \dfrac{K(2Q)}{(2R)^2} - \dfrac{KQ}{(4R)^2} = \dfrac{7KQ}{16R^2}$ towards O

12.(A)



$\left(\text{using } V(r,\theta) = \dfrac{p\cos\theta}{4\pi\in_0 r^2}\right)$

$\Rightarrow$ net potential at $\left(\dfrac{r}{2}, \dfrac{\sqrt{3}r}{2}, 0\right)$ is $\dfrac{p\cos30^\circ}{4\pi\in_0 r^2} + \dfrac{p\cos60^\circ}{4\pi\in_0 r^3}$

$$= \frac{\sqrt{3}p}{2\left(4\pi\in_0 r^2\right)} + \frac{p\cos60^\circ}{4\pi\in_0 r^2} = \frac{p\left(\sqrt{3}+1\right)}{8\pi\in_0 r^2}$$

13.(B) $f = \dfrac{Kq_1q_2}{4\pi\in r^2}$ and $\dfrac{f}{3} = \dfrac{K(q_1-q_2)^2}{4\pi\in_0 r^2 4}$ where q_1 and q_2 be initial charges.

$\Rightarrow \quad 4q_1q_2 = 3q_1^2 + 3q_2^2 - 6q_1q_2$; Let $\dfrac{q_1}{q_2} = x \Rightarrow 3x^2 - 10x + 3 = 0 \Rightarrow x = 3$ or $\dfrac{1}{3}$

14.(AC) If forces due to both the fields cancel out then (A). If forces due to both fields are along the same line and the particle is given the initial velocity along the same line, then also (A).
In any other case, the net force acting on the particle will be uniform. From basic 2-D motion, we know that if acceleration is uniform but not parallel to initial velocity, the trajectory of the particle is a parabola.

15.(ABD) For (A), use the fact that potential at center of sphere, due to Q_2 will be exactly cancelled by charge $-Q_2$ induced on the surface of the cavity. Hence (A).
Potential at any point at surface of cavity is equal to potential at the centre of sphere. Also, electric field inside cavity is known. So, potential at any general point inside cavity can be found be performing the line integral of electric field from the surface of cavity upto that point. This will solve (B).
Potential outside the sphere cannot be found easily as the charge on the outer surface will be non-uniform. Hence C is incorrect as the given answer for uniform $Q_2 + Q_3$.
(D)can be calculated by making the net potential at the center of sphere zero and finding new charge on sphere.

16.(AB) $x_0 =$ deformation in equilibrium state.

$$2Kx_0 = \frac{\sigma}{\epsilon_0} \cdot q \quad \therefore \quad x_0 = \frac{\sigma q}{2K \epsilon_0}$$

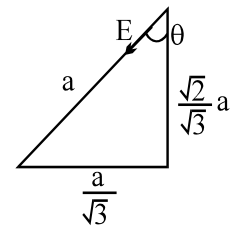
Springs are connected in parallel $K_{eq} = 2K$ Angular frequency $= \sqrt{\frac{2K}{m}}$

17.(AD) We know that for constant r, electric field due to a dipole is maximum at axial point and minimum at equatorial point. Magnitude of electric field continuously decreases from $2KP/r^3$ to KP/r^3 as we move from an axial point to an equatorial point, keeping r constant. In the present case, if E = 10N/C at any point, then it cannot be less than 5N/C or more than 20N/C at any point.

18.(BCD)
(A) is incorrect by conservation of charge.
(B) is correct. The net field inside the sphere is zero only because the electric field due to induced charges cancels the external electric field at all points inside the sphere.
(C) is a basic property of conductors
(D) As stated in part (B), the electric field lines are eliminated within the spherical region.

19.(BC) $E = \frac{kq}{a^2} \Rightarrow E_{net} = 3E\cos\theta = 3 \times \frac{kq}{a^2}\frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}kq}{a}$ (B)

$V = \frac{3kq}{a}$ (C)

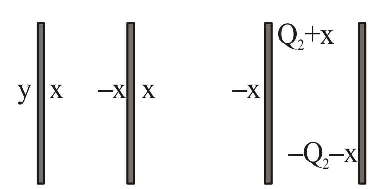


20.(AC) Potential due to a uniformly charged spherical shell is given by:

$$V = \frac{KQ}{r}\ for\ r > R \quad \text{and} \quad V = \frac{KQ}{R}\ for\ r < R$$

21.(AB) $xa + xb + (Q_2 + x)c = 0 \Rightarrow x = \frac{-Q_2 c}{a+b+c}$

$Q_1 = y + x + y - Q_2 - x \Rightarrow y = \frac{Q_1 + Q_2}{2}$



Potential different between A & B is $V = \frac{Q_2 ca}{(a+b+c)S\varepsilon_0}$.

Similarly, p.d between C & D depends upon Q_2

22.(B) In part B, difference is due to the mass of electrons. Part (C) will not be true for a curved electric field line.

23.(ABD) $E_x = 10V/m$, $E \geq E_x$

24.(ABCD) (A) Net charge enclosed by Gaussian surface is zero.
(B) Net charge enclosed by Gaussian surface is zero.
(C) As the change is displaced towards sphere amount of negative charge induced on right half of sphere will increase hence net flux through right hemispherical closed Gaussian surface increases.
(D) Same reason as (C) and hence charge distribution on outer surface of sphere will change.

25.(ABC) $V_A = V_B \Rightarrow$ work $= 0$

26.(ABC)
If moved slightly along x-direction, say towards left, the attractive force of left wire will be more than the attractive force of the right wire. Hence equilibrium will be unstable in this direction.
If moved in y-direction, the force will remain zero.
If moved in z-direction, electron will be attracted back towards the wires.

27.(ABCD) Charge on $a_1 = (r_1\ \theta)\ \lambda$ and Charge on $a_2 = (r_2\ \theta)\ \lambda$

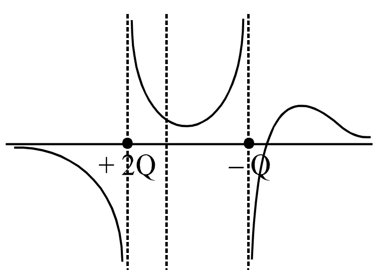
Ratio of charges $= \frac{r_1}{r_2}$

E_1, Field produced by $a_1 = \frac{K[(r_1\ \theta)\lambda]}{r_1^2} = \frac{KQ\lambda}{r_1}$; E_2, Field produced by $a_2 = \frac{KQ\lambda}{r_2}$

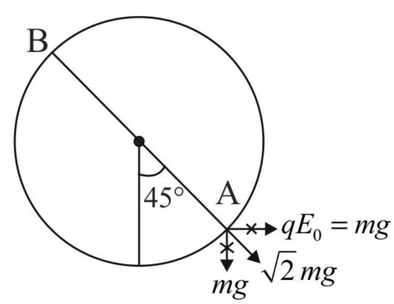
as $r_2 > r_1$
Therefore $E_1 > E_2$ i.e. Net field at A is towards a_2.

$V_1 = \frac{K.(r_1\theta)\lambda}{r_1} = K\theta\ \lambda$, $V_1 = \frac{k(r_2\theta)\lambda}{r_2} = k\theta\lambda$ $\therefore$ $V_1 = V_2$

28.(AD)



29.(BC) Position A is position of equilibrium and B is critical position so tension and speed during motion is maximum at A and minimum at B.



$V_{MIN} = V_B = \sqrt{g_{eff}\ell} = \sqrt{\sqrt{2}g\ell}$

$V_{MAX} = V_A = \sqrt{5g_{eff}\ell} = \sqrt{5\sqrt{2}g\ell}$

$T_{MAX} = T_{MIN} = 6mg_{eff} = 6\sqrt{2}mg$

30.(AD) By conserving energy of the first ball,

$$0+\frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{r_{12}}+\frac{1}{r_{13}}+\frac{1}{r_{14}}+.....+\frac{1}{r_{1N}}\right]=K_1+0$$

And for the second ball,

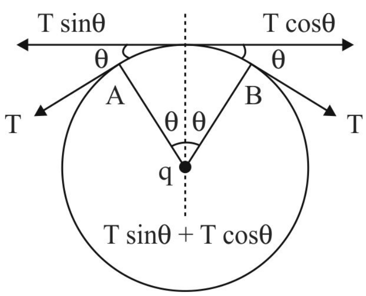
$$0+\frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{r_{23}}+\frac{1}{r_{24}}+.....+\frac{1}{r_{2N}}\right]=K_2+0$$ [It can be observed $K_1 > K_2$]

$\Rightarrow \quad K = K_1 - K_2 = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{d}\right) \quad \Rightarrow \quad q=\sqrt{4\pi\varepsilon_0 dK}$

31.(ABCD) Flux through two circular surfaces is

$$\phi = 2\times\frac{q}{2\varepsilon_0}\left[1-\frac{4R}{\sqrt{(3R)^2+(4R)^2}}\right]=\frac{q}{5\varepsilon_0} \quad ; \quad \phi_{\text{curved}}=\frac{q}{\varepsilon_0}-\frac{q}{5\varepsilon_0}=\frac{4}{5}\frac{q}{\varepsilon_0}$$

32.(AB) Consider a small element AB, θ is very small. Then



$AB = R(2\theta)$

Change on AB is $dQ=\frac{Q}{2\pi R}(2R\theta)=\frac{Q\theta}{\pi}$

$$2T\sin\theta=\frac{dQ.q}{4\pi\epsilon_0 R^2}=\frac{Qq\theta}{4\pi^2\epsilon_0 R^2}$$

$$2T=\frac{Qq\theta}{4\pi\epsilon_0 R^2} \text{ or } T=\frac{Qq}{8\pi^2\epsilon_0 R^2}$$

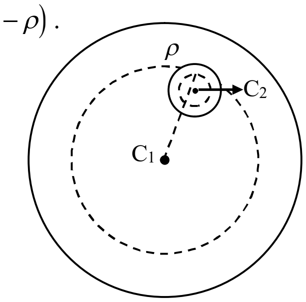
33 (ACD) At steady state electric force is balanced by centrifugal force.

At r radial distance

$M\omega^2 r^2 = -eE\vec{r}$

Now integrate $E=\frac{-dV}{dr}$ and get potential at centre $\frac{m\omega^2 R^2}{2e}$

34.(A) Let us assume that the cavity is filled with equal and opp. charge density $(+\rho \text{ and } -\rho)$.



$\overline{E}_P = \frac{\rho}{2\varepsilon_0}\left(\overline{C_1P}+\overline{PC_2}\right)=\frac{\rho\overrightarrow{C_1C_2}}{2\varepsilon_0}$ i.e. parallel to line joining C_1 and C_2

35.(A) $|\overline{E}_P| = \frac{\rho}{2\varepsilon_0}|\overrightarrow{C_1C_2}| = \frac{\rho a}{2\varepsilon_0}$

For Q. 36 – 39

Let us consider forces acting on bead P as shown in Figure. These forces are :

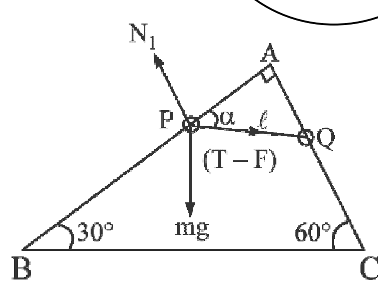
(i) Weight mg vertically downwards

(ii) Tension T in the string

(iii) Electric force between P and Q given by

$$F=\frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{\ell^2}$$

(iv) Normal reaction N_1



The net force along the string is $(T - F)$. Bead P will be in equilibrium, if the net force acting on it is zero. Resolving forces mg and $(T-F)$ parallel and perpendicular to plane AB, we get, when the bead P is in equilibrium,

$mg \cos 60° = (T-F) \cos\alpha$ . . . .(i)

and $N_1 = mg\cos 30° + (T-F) \sin\alpha$ . . . . (ii)

For the bead at Q, we have

$mg \sin 60° = (T - F) \sin\alpha$ . . . . (iii)

and $N_2 = mg \cos 60° + (T-F) \cos\alpha$ . . . . (iv)

36.(C) Dividing Eq. (iii) by Eq. (1), we get

$\tan\alpha = \tan 60°$ *or* $\alpha = 60°$ which is choice (C)

37.(C) Using $\alpha = 60°$ in (iii), we have

$mg\sin 60° = (T - F)\sin 60°$ Or $T - F + mg = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{\ell^2} + mg$ . . . . .(v)

So the correct choice (C)

38.(A) From Eq. (iv) we have

$N_2 = mg \cos 60° + mg\cos 60° = mg$; Thus is the correct choice is (A)

39.(D) When the string is cut, $T = 0$. Putting . Putting $T = 0$ is Eq. (v), we get

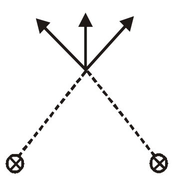
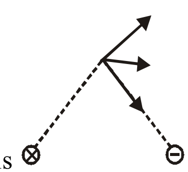
$$mg = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{\ell^2}$$

The right hand side of this equation should be positive which is possible if q_1 and q_2 have opposite signs. Thus, for equilibrium the beads must have unlike charges. The magnitude of the product of the charges is

$|q_1 q_2| = (4\pi\varepsilon_0) mg\ell^2$,

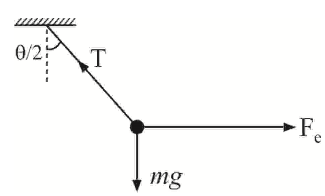
which is choice (D)

40. **[A – q,r ; B–- p,s,t ; C- p,q,t ; D- p,s,t]** v = $\frac{kQ}{r}$ & E = $\frac{kQ}{r^2}$ & U = $\frac{kq_1q_2}{r}$

for by symmetry  is | | to the axis  is ⊥ as to the axis.

41. **[A – pq ; B - s ; C- s ; D – r]**

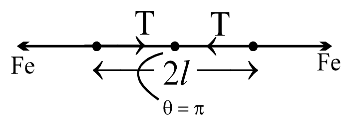
For small 'θ' separation of balls = $l\theta$ and hence, T = mg



$$T\sin\frac{\theta}{2} = T\frac{\theta}{2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(l\theta)^2} \Rightarrow q^2 \propto \theta^3,\ q \propto \ell,\ T \propto \ell^{-2}$$

In satellite

$$T = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(2l)^2}\ (g_{eff} = 0) \Rightarrow T \propto \ell^{-2},\ \theta = \pi$$



42. **[A– q ; B – p ; C- s ; D- r]**

$V_0 = \dfrac{Kq}{R/2} + \dfrac{K(-2q)}{2R} = \dfrac{Kq}{R}$ (Q, R) $\quad$ V_0 (due to inner surface charge) $= -\dfrac{Kq}{R}$ (P, R)

43.(9) When outer surface is grounded charge '–Q' resides on the inner surface of sphere 'B'
Now sphere A is connected to earth potential on its surface becomes zero.
Let the charge on the surface A becomes q

$$\frac{kq}{a} - \frac{kQ}{b} = 0 \quad \Rightarrow \quad q = \frac{a}{b}Q$$

Consider the figure. In this position energy stored

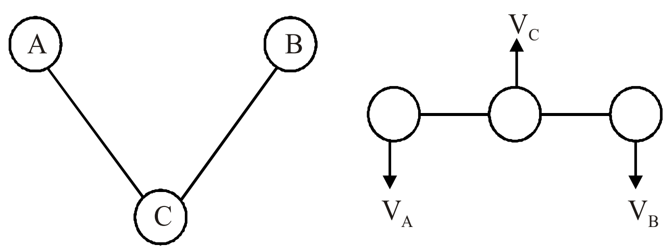
$$E_1 = \frac{1}{8\pi\varepsilon_0 a}\left[\frac{a}{b}Q\right]^2 + \frac{Q^2}{8\pi\varepsilon_0 b} + \frac{1}{4\pi\varepsilon_0 b}\left[\frac{a}{b}Q\right](-Q)$$

when 'S_3' is closed, total charge will appear on the outer surface of shell 'B'. In this position energy stored

$$E_2 = \frac{1}{8\pi\varepsilon_0 b}\left(\frac{a}{b} - 1\right)^2 Q^2$$

Heat produced (Q) $= E_1 - E_2 = \dfrac{Q^2 a(b-a)}{8\pi\varepsilon_0 b^3} = 1.8$ $\quad$ So, $\quad 5Q = 5\times 1.8 = 9$

44.(2) Takes all three bodies as system electrostatic force
(1) Apply law of conservation of momentum in direction y $\quad$ (2) Apply law of conservation of energy,
So $\quad m_A\vec{v}_A + m_B\vec{v}_B + m_C\vec{v}_C = 0 \Rightarrow v_C = -2v_A$ (as $V_A = V_B$)



Change in electrostatic P.E. = Increase in KE

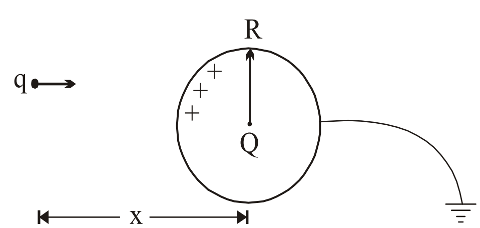
$\dfrac{kQ^2}{\ell} - \dfrac{kQ^2}{2\ell} = \dfrac{1}{2}m_A v_A^2 + \dfrac{1}{2}m_B v_B^2 + \dfrac{1}{2}m_C v_C^2$ $\quad$ [$v_A = v_B$, $V_C = -2v_A$; $\quad$ So $V_C = 2$ m/s

45.(4) $\vec{E} = \dfrac{a\hat{r}}{r^2} \quad \therefore \quad dV = -\vec{E}\cdot d\vec{r} \quad \Rightarrow \quad \Delta V = a\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right)$

$$\Delta V = \frac{a}{r_2} - \frac{a}{r_1} = \frac{a}{5} - \frac{a}{7} = \frac{2a}{35} = 2 \times \frac{280}{35} = 16 \text{ V}$$

46.(2) $\dfrac{kq}{x} + \dfrac{kQ}{R} = 0 \quad \Rightarrow \quad Q = -\dfrac{qR}{x}$

$$\frac{dQ}{dt} = \frac{qR}{x^2}\left(\frac{dx}{dt}\right) = \frac{10^{-3}\times 12}{2\times 2} = 0.5 \times 10^{-3} = 0.5 \times 10^{-6}$$



47.(3) Net Electric field t $\vec{P}$
$\vec{E} = \vec{E}_1 + \vec{E}_2$ due to vertical wire

$\vec{E}_1 = \dfrac{\lambda_1}{2\pi\varepsilon_0 y}$ in + y direction , $\vec{E}_2 = \dfrac{\lambda_2}{2\pi\varepsilon_0 x}$ due to vertical wire in x direction.

α angle of electric field with x direction $\tan\alpha = \dfrac{E_1}{E_2} = \dfrac{\lambda_1 / 2\pi\varepsilon_0 y}{\lambda_2 / 2\pi\varepsilon_0 x} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{\lambda_1 x}{\lambda_2 y} \Rightarrow \dfrac{\lambda_1}{\lambda_2} = \dfrac{x}{y}\sqrt{3}$; $\dfrac{\lambda_1}{\lambda_2} = 3$

48.(9) In uniform electric in vertical direction if (+ve) charge feels extra acceleration in downward direction, then (–ve) charge will feel acceleration in upward direction.

$v_{uncharged} = 5\sqrt{5}\ m/\sec$; $v = 0$, h = height

$v^2 - u^2 = -2(g)h$; $-(5\sqrt{5})^2 = -2gh$

$u_{q+} = 13$ m/sec ; $v = 0$, $h = h$

$v^2 - u^2 = 2\left(g + \dfrac{F_E}{m}\right)h$; $0 - (13)^2 = -2\left(g + \dfrac{F_E}{m}\right)h$

Let $u_{q-} = u$(say) ; $v = 0$ h = ht

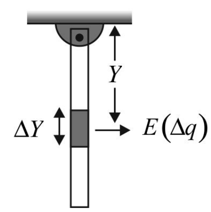
$v^2 - u^2 = -2\left(g - \dfrac{F_E}{m}\right)h \Rightarrow -u^2 = -2\left(g - \dfrac{F_E}{m}\right)h$; u = 9 m/sec

49.(6) At a distance r from the center, take a thin shell of thickness dr. A charge $dq = \rho dv$ will be trapped inside this shell. Now apply Gauss' law on this volume (dv) to obtain the required result.

50.(1) If it was complete sphere, then total $flux = \dfrac{Q}{\epsilon_0}$

$\Rightarrow$ if position is changed, then $flux$ through the two hemisphere is also interchanged $\Rightarrow \alpha = 1$

51.(3) $\tau_{Net} = \int_0^L YE(dq) = \int_0^L YE\lambda(dY) = \lambda E\int_0^L Y\, dY$



$\tau_{Net} = \dfrac{\lambda E L^2}{2}$

$\alpha = \dfrac{\tau_{Net}}{I} = \dfrac{\lambda 3EL^2}{2ML^2} = \dfrac{3E\lambda}{2M}$

52.(5) Assume 'ρ' and '$-\rho$' in the cavity then

$$V_\rho = \frac{3}{2}\frac{K}{R}\left(\rho.\frac{4}{3}\pi R^3\right)$$

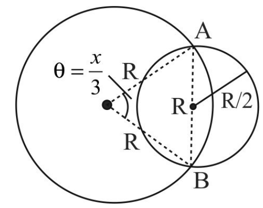
$$V_{-\rho} = \frac{K\left[-\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right]}{\frac{R}{2}}$$

$$V_C = V_\rho + V_{-\rho} = 2\pi K\rho R^2 - \frac{\pi K\rho R^2}{3} = \frac{5\pi K\rho R^2}{3}$$

$$V = \frac{5\rho R^2}{12\epsilon_0}$$

53.(0.75) $\phi = \frac{q_{in}}{\varepsilon_0} = \frac{\int dq}{\varepsilon_0} = \frac{\int A\,dx\rho}{\varepsilon_0} = \frac{A}{\varepsilon_0}\int \rho dx = \frac{1}{\varepsilon_0}$ [area under the curve] $= \frac{3}{4}$

54.(2) Flux will be maximum when maximum length of ring is inside the sphere.



This will occur when the chord AB is maximum. Now maximum length of chord AB= diameter of sphere. In this case the arc of ring inside the sphere subtends an angle of $\frac{\pi}{3}$ at the centre of ring.

$\therefore$ charge on this arc $= \frac{R\pi}{3}.\lambda$ $\therefore$ $\phi = \frac{\frac{R\pi}{3}\lambda}{\epsilon_0} = \frac{R\pi\lambda}{3\epsilon_0}$ Solving ans 2

55.(1) $\oint \vec{E}.d\vec{A} = \frac{\int \rho dV}{\epsilon_0}$

$E \times 4\pi r^2 = \frac{k\int r^n \times 4\pi r^2 dr}{\epsilon_0} = \frac{4\pi k}{\epsilon_0}\frac{r^{x+3}}{x+3}$; $E = \frac{k}{(n+3)\epsilon_0}\left(r^{n+1}\right)$; $n+1=2$

$\Rightarrow n = 1$

56.(2.5) In the remaining three quadrants, put three more quarter sheets to convert this given arrangement to that of infinite sheet. Now contribution from all the four quarters to the z-component will be same. Hence due to a quarter E.F. at point (0, 0, z) will be, $\vec{E} = \frac{1}{4}\left(\frac{\sigma}{2\epsilon_0}\right)\frac{|z|}{z}\hat{k} = \frac{\sigma}{8\epsilon_0}\frac{|z|}{z}\hat{k}$.

Hence potential difference between points (0, 0,1) and (0, 0, 2) will be,

$v_{2d} - v_d = -\int_d^{2d} \vec{E}.d\vec{l}$; $d\vec{l} = dz\hat{k}; \vec{E} = \frac{\sigma}{8\epsilon_0}\frac{|z|}{z}\hat{k}$

$v_{2d} - v_d = -\int_d^{2d} \frac{\sigma}{8\epsilon_0}\frac{|z|}{z}\hat{k}.dz\,\hat{k} = -\frac{\sigma}{8\epsilon_0}\int_d^{2d}\frac{|z|}{z}dz$; $v_d - v_{2d} = \frac{\sigma}{8\epsilon_0}|d|$

Substitute the value of σ and d

57.(9) The value of flux is maximum through surface AB GH, because charge in front of this surface is maximum, and flux is $= \frac{9q}{24\varepsilon_0}$.

58.(4) Work done by magnetic force is zero so from work energy theorem

$\frac{1}{2}m_p v_B^2 = \frac{1}{2}m_p v_A^2 + q\Delta v$

and simultaneously there is no change of velocity component along the direction of perpendicular to electric.

$v_A \sin\alpha = v_B \sin\beta$

After solving $\Delta v = 16$ Volt

59.(20) For maximum angular velocity, rotation is equal to 90º.

$W_{EF} = \Delta KE$

$(qE)R = \frac{1}{2}(mR^2)\varepsilon^2$; $\omega = \sqrt{\frac{2(q)E}{mR}} = \sqrt{\frac{2\left(\frac{Ql}{2\pi R}\right)}{mR}} = \sqrt{\frac{QlE}{\pi m R^2}}$ substituting values Ans is 20

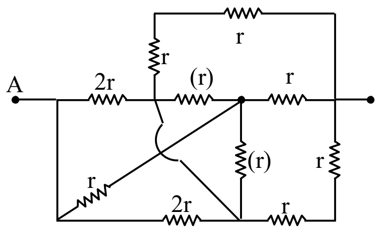
60.(4) The force experienced by an electric dipole placed in a non-uniform electric field is given by, $F = p\frac{\partial E}{\partial l}$ where $\frac{\partial E}{\partial l}$ is directional derivative of $\vec{E}$ along the dipole moment. Here, dipole is placed along x-axis, so $\frac{\partial E}{\partial l}$ corresponding to component of $\frac{\partial E}{\partial u}$ is along x-axis.

$$\vec{E} = p\frac{\partial \vec{E}}{\partial x} = 6x\hat{i} + 6y\hat{i} + 0\hat{k} \quad \Rightarrow \quad \vec{F} = |\vec{p}|\left(\frac{\partial \vec{E}}{\partial x}\right)_{x \text{ component}} = p(6\times 4\hat{i} + 6\times 0\,\hat{j}) \Rightarrow \quad \vec{F} = 24p\hat{i}$$

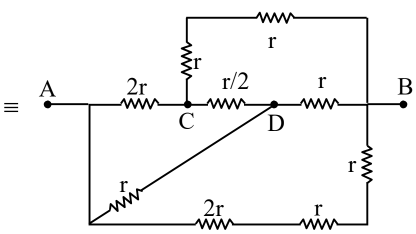
DC CIRCUITS AND CAPACITORS

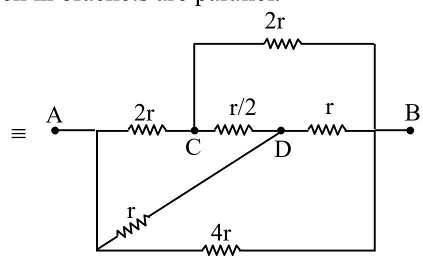
1.(D) $(1680 + r)I = 20 \Rightarrow (2930 + r)I = 30 \Rightarrow 2 \times 2930 + 2r = 3 \times 1680 + 3r \Rightarrow r = 820, \quad I = 8mA$

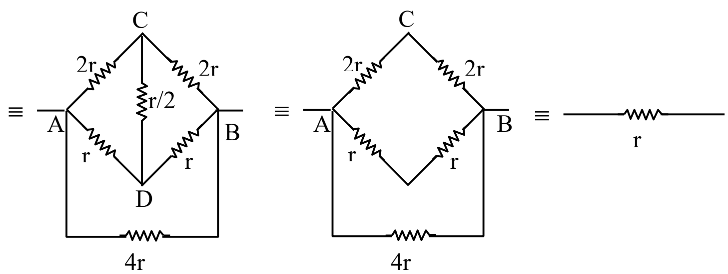
2.(D)



Resistors whose resistances are written in brackets are parallel.







3.(A) $E = j\rho$ [j = current density]

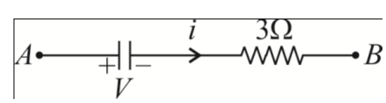
$j = \dfrac{I}{\pi r^2}$ [r = radius of cross section at distance 'x' from left end]

$r = [a + \dfrac{(b-a)}{l}x]$ Hence, $E = \dfrac{Vl^2\rho}{\pi R(al + (b-a)x)^2}$

4.(A) $5 \times 10^{-3} A = \dfrac{I(0.1\Omega)}{(0.2 + 0.3 + 0.5)} \Rightarrow I = 50mA$

5.(C) $V_A - V_B =$ voltage drop across capacitor + voltage drop across resistor

$\therefore \quad -11 = \dfrac{Q}{C} + iR \Rightarrow -11 = \dfrac{16}{4} + i.3\times10^3 \Rightarrow i = -5mA$



Power delivered by capacitor $P = iV = (5mA)(4V) = 20mW$

6.(D) Initial charging rate = initial current in the line of capacitor = $\dfrac{2E}{5R}$

Steady state p.d. across capacitor :

$V_0 = \dfrac{2}{3}E \Rightarrow q_0 = CV_0 = \dfrac{2}{3}EC \Rightarrow t = \dfrac{q_0}{i} = \dfrac{\frac{2}{3}EC}{\frac{2E}{5R}} = \dfrac{5}{3}RC$

7.(A) Suppose that the inner sphere is at a higher potential than outer sphere. Let the current be i.
Consider a thin shell of thickness dr at a distance r from the centre. Let voltage across it be dV. Then, applying

$V = iR$

For thin shell: $dV = i\cdot\dfrac{dr}{\sigma 4\pi r^2} \;\Rightarrow\; Edr = i\cdot\dfrac{dr}{4\pi r^2}\cdot\dfrac{1}{kE}$

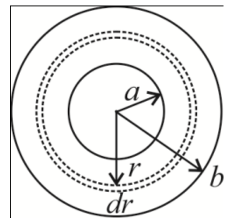
$\therefore \quad E = \sqrt{\dfrac{i}{4\pi k r^2}} = \dfrac{C}{r}$ where $C = \sqrt{\dfrac{i}{4\pi k}}$

Using, $E = -\dfrac{dV}{dr}$

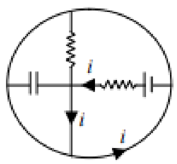
$\dfrac{C}{r} = -\dfrac{dV}{dr} \quad \Rightarrow \quad C(\ell n r)_a^b = V_a - V_b$

$V = V_a - V_b = C\ln(b/a)$

$V = \sqrt{\dfrac{i}{4\pi k}}\,\ell n(b/a) \quad \Rightarrow \quad i = \dfrac{V^2 4\pi k}{[\ell np(b/a)]^2}$



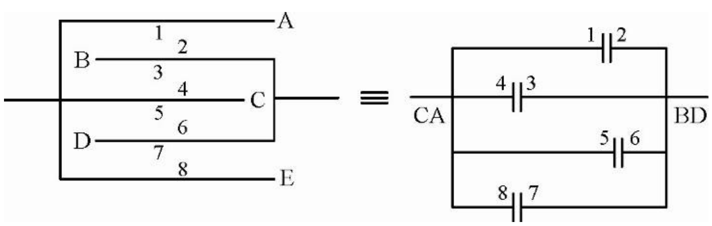
8.(D) $\Rightarrow$ P.d. across C is zero $\Rightarrow$ charge $= 0$



9.(D) Let n_1 : no. of capacitors be connected in parallel, n_2 : no. of such parallel combination connected in series.

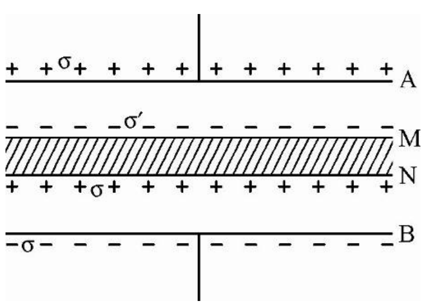
$n_2 = \dfrac{1000}{250} = 4$ and $\dfrac{n_1(8\,\mu F)}{n_2} = 16\,\mu F \Rightarrow n_1 = 8 \quad \Rightarrow$ Total no. of capacitors required = 32

10.(B)



Charge on C = sum of charges at 4&5 = 2 CV $= 2\times 2\times 10 = 40\ \mu C$

11.(B)



$\Rightarrow$ inside dielectric, field $= \dfrac{\sigma}{\in\in_0} = \dfrac{\sigma - \sigma'}{\in_0} \quad \Rightarrow \quad \sigma' = \left(\dfrac{\in - 1}{\in}\right)\sigma$

$\Rightarrow$ Force between A & M $= \left(\dfrac{\in - 1}{\in}\right)\sigma A\,.\,\dfrac{\sigma}{2\in_0}$

(Using Eq = F) $= \dfrac{\sigma^2(\in - 1)A}{2\in\in_0}$

12.(B) Current is obviously constant, by charge conservation. Using $i = n_0 eAv_d$, we can say that if A is non-uniform, v_d will be non-uniform. Similarly, since v_d depends on electric field, electric field will also be non uniform.

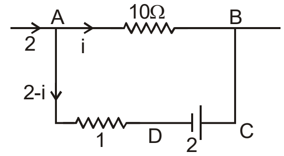
13.(ABC) Let the point where jockey touches wire be called S. Then the direction of current shown in figure indicates that voltage across QS is less that E_2. This can happen if:

1. E_1 is too low
2. r is too high (if r is too high, it will take up more voltage and less will be left for QS).

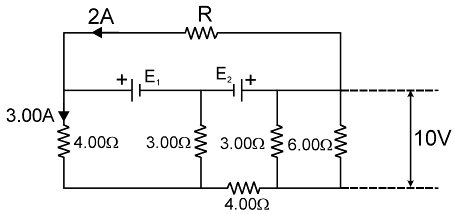
14.(BD) Let the currents be as shown in the figure:

KVL along ABCDA $\Rightarrow -10i - 2 + (2-i)1 = 0 \therefore i = 0$

Potential difference across S = $(2 - i)1 = 2 \times 1 = 2\ V$.



15.(ABCD)



After redrawing the circuit

(a) $I_4 = 5A$

(b) From loop (1)

$-8(3) + E_1 - 4(3) = 0 \Rightarrow E_1 = 36$ volt

(c) From loop (2)

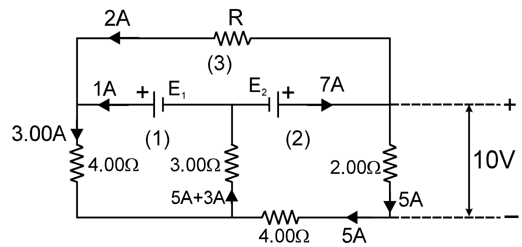
$+4(5) + 5(2) - E_2 + 8(3) = 0$

$E_2 = 54$ volt

(d) From loop (3)

$-2R - E_1 + E_2 = 0$

$$R = \frac{E_2 - E_1}{2} = \frac{54 - 36}{2} = 9\ \Omega$$

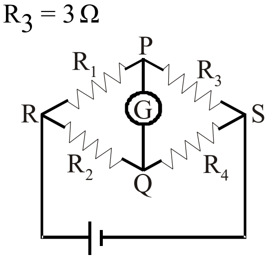


16.(AC) Equivalent diagram is as shown. If P is moved 2cm right, then $R_1 = 12\,\Omega$, $R_3 = 3\,\Omega$

$\frac{R_1}{R_3} = \frac{R_2}{R_4}$ (Hence wheat stone will be balanced.)

If S is moved left by $\frac{5}{3}$ cm, then $R_3 = \frac{10}{3}$ and

$R_4 = \frac{20}{3}$ hence $\frac{R_1}{R_3} = \frac{R_2}{R_4}$ (hence wheatstone's bridge will be balanced.)



17.(AB) $\frac{R}{AC} = \frac{R_1 + R_2}{CB} \Rightarrow \frac{R}{L/4} = \frac{R_1 + R_2}{3L/4} \Rightarrow R_1 + R_2 = 3R$ and $\frac{R + R_1}{2\ell/3} = \frac{R_2}{\ell/3} \Rightarrow R + R_1 = 2R_2$

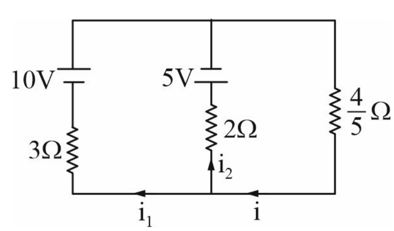
$R_2 = \frac{4R}{3}$ and $R_1 = \frac{5R}{3}$

18. (AC) $\dfrac{\frac{10}{3} - \frac{5}{2}}{\frac{1}{3} + \frac{1}{2}} = \varepsilon_{eq}$

$\frac{5/6}{5/6} = 1\text{V}$

$r_{eq} = \frac{6}{5} \Rightarrow i = \dfrac{1}{\frac{4}{5} + \frac{6}{5}} = 0.5\text{A}$

P.d. = $0.5 \times \frac{4}{5} = 0.4\text{ V}$; $0.5 \times \frac{4}{5} = 0.4\text{ V}$



$-3i_1 + 10 - 1.0 - 0.4 = 0 \quad \Rightarrow \quad i_1 = +ve$

$-0.4 - 2i_2 - 5 = 0 \quad \Rightarrow \quad i_2 = -ve$

19.(BD)

20.(ABC) $\because$ B_2 and B_3 are in series, $\therefore$ $i_2 = i_3$;

Also, $V_2 + V_3 = V_1$

$\because \quad P_1 = P_2 = P_3 = P_4 \Rightarrow P_2 = P_3 \text{ or } i_2^2 R_2 = i_2^2 R_3 \Rightarrow R_2 = R_3$

$\therefore \quad V_2 = V_3 = \frac{V_1}{2} \Rightarrow \frac{V_1^2}{R_1} = \frac{V_1^2}{R_1} = \frac{V_1^2}{36}$ and $P_3 = \frac{V_3^2}{R_3} = \frac{V_1^2}{4R_3}$

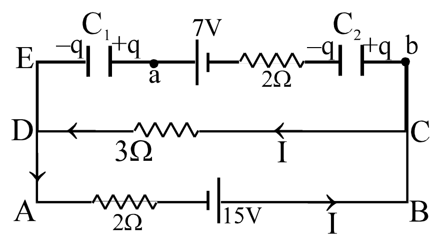
$\frac{V_1^2}{36} = \frac{V^2}{4R_3} \Rightarrow R_3 = 9\Omega = R$

$P_1 = P_4 \Rightarrow I_1^2 R_1 = I_4^2 R_4$ or $\left(\frac{I_4}{3}\right)^2 \times 36 = I_4^2 R_4 \Rightarrow R_4 = R\Omega$

Voltage output of battery = $V_1 + V_4$

$\left[P_1 = 4 = \frac{V_1^2}{36} \Rightarrow V_1 = 12V \ \& \ P_2 = 4 = \frac{V_4^2}{4} \Rightarrow V_4 = 4V\right] \quad \therefore \quad V_1 + V_4 = 16V$

21.(ACD)



At steady state : $I(3) + I(2) = 15$

$I = 3$

KVL $C \to D \to E \to a \to b \to C$

$(V/C) - I(3) + \frac{q}{11} - 7 + \frac{q}{5} = 0$

$\Rightarrow \quad \frac{q}{11} + \frac{q}{5} = 7 + 3 \times 3 = 16 \quad \Rightarrow \quad q = 55\ \mu C$

$KVL : a \to b \quad \Rightarrow \quad V_a - 7 + \frac{q}{5} = V_b \Rightarrow V_a - V_b = 7 - \frac{q}{5} = 7 - \frac{55}{5} = -4V$

$P.d.$ across $C_1 \quad \Rightarrow \quad \frac{q}{11} = \frac{55}{11} = 5V$; $\quad P.d.$ across $C_2 \quad \Rightarrow \quad \frac{q}{5} = 11V$

P.d. across terminal = $15 - I(2) = 15 - 3 \times 2 = 9V$

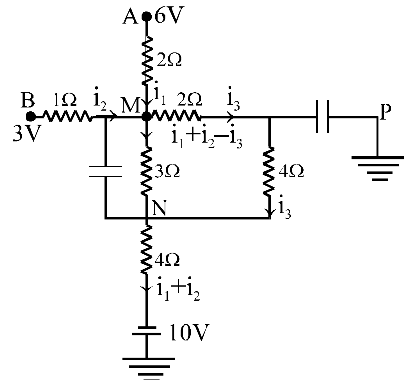
22.(AB) We know that the capacitance of an empty capacitor increases k times if a dielectric is inserted in it. Therefore, in this case, the capacitance of combination will increase upon insertion of a dielectric. Also, by Q = CV, charge supplied by battery also proportionately increases for keeping V constant.

23.(ABC) At t = 0, capacitor will have no voltage across it. Hence A. Voltage across capacitor will gradually increase with time. Hence B. C can be calculated by the usual charging equation for capacitor.

24.(ABC) $2i_1 + 3(i_1 + i_2 - i_3) + 4(i_1 + i_2) - 10 = 6$

$i_2 + 2i_3 = 3$

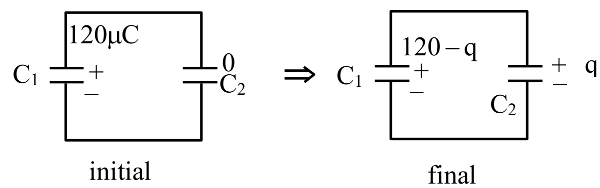
$3(i_1 + i_2 - i_3) - 6i_3 = 0$



25.(AC) Charge flown through battery = $C_1V = 6 \times 20 = 120\mu C$

After closing S_2, common potential $V_C = \dfrac{120\mu C}{9\mu F}$

Final $C_2 = C_2V_C = 3 \times \dfrac{120}{9} = 40\mu C$

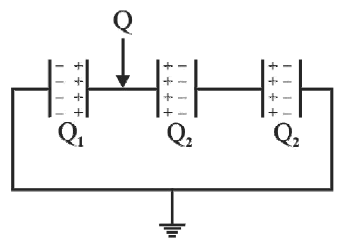


Heat produced $= \left[\dfrac{120^2}{2 \times 6}\right] - \left[\dfrac{80^2}{2 \times 6} + \dfrac{40^2}{2 \times 6}\right]\mu J \simeq 0.53\ mJ$

26.(ABCD) $Q_1 + Q_2 = Q$ and $\dfrac{Q_1}{C} = \dfrac{2Q_2}{C}$

$Q_1 = 2Q_2$; $Q_1 = \dfrac{2Q}{3}$

$Q_2 = \dfrac{Q}{3}$ and potential at point A is $\dfrac{2Q}{3C}$



27.(ACD) $i = \dfrac{E}{5R}$

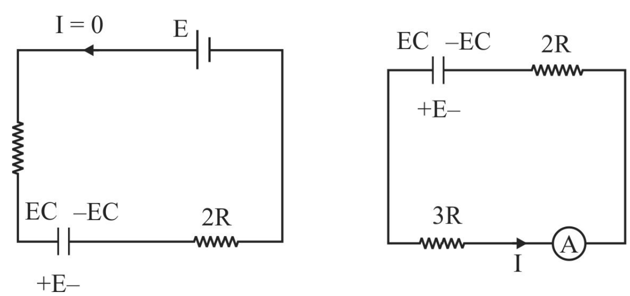
Equation of charging of capacitor

$q = CEe^{-t/\tau}$

$q = ECe^{-t/5RC}$

at t = 5 RC $ln2$

$q = \dfrac{EC}{2}$



$\Delta U_{cap} = H_{2R} + H_{3R}$

$\dfrac{Q^2}{2C} - \dfrac{(Q/2)^2}{2C} = H_{2R} + H_{3R}$

$\dfrac{3Q^2}{8C} = H_{2R} + H_{3R}$

($\because H = i^2R$ and i is same in series, $\therefore$ $H \propto R$)

$H_{3R} = \dfrac{3}{5} \times \dfrac{3Q^2}{8C} = \dfrac{9Q^2}{40C}$

$= \frac{9}{40} CE^2$

28.(ABCD) Uncharged capacitor behave like zero resistance

$I = \frac{36}{\text{Req}} = \frac{36}{3} = 12 \text{ amp}$

$I_C = \frac{6}{9} \times 12 = 8 \text{ amp}$

In steady state, capacitor behave like a large resistance

$I = \frac{36}{4} = 9 \text{ amp}$

29.(ABCD) Initially capacitor behave like zero resistance.

$$P_{\text{initially}} = \frac{V_0^2}{\frac{2}{3}\left(\frac{V_0^2}{P_0}\right)} = \frac{3}{2} P_0$$

Finally capacitor behaves like large resistance.

$$P_{\text{final}} = \frac{V_0^2}{2\left(\frac{V_0^2}{P_0}\right)} = \frac{P_0}{2}$$

30.(ABCD) Potential difference across two plates must be equal to ε.

So potential difference across any point on plate-1 and any point on plate-2 will be constant

So electric field will be same.

31.(ABD) As $Q_1 = Q_2$ or $C_1E_1 = C_2E_2$ Hence **(A)** is correct.

$q = Q_0 e^{\frac{-t}{RC}}$ $\therefore$ Slope $\frac{dq}{dt} = Q_0 e^{\frac{-t}{RC}} \times \frac{-1}{RC}$ $\Rightarrow$ Slope $\propto \frac{-1}{RC}$

$\therefore$ $R_1C_1 > R_2C_2$ **(B)** is correct

$R_1C_1 > R_2C_2$ but C_1 and C_2 are not known hence **(C)** is incorrect As $Q_1 = Q_2$ Hence **(D)** is correct.

32.(ABCD) By energy conservation A is correct.

B is correct as Current is max at t = 0 . Just before steady state voltage across C is same as battery hence C is correct.

$\therefore$ ABCD

33.(ABC) Initial charge (before filling the dielectric slab) = 10 × 10 = 100 μC

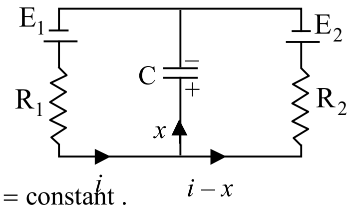
Final charge (after filling the dielectric slab) = 10 × 30 = 300 μC

$\therefore$ Increase in charge = 200μ*C*.

34.(A) Reading of $C = V$ {i in that branch = 0} ; Reading of $A = \frac{V}{R}$ $\Rightarrow$ Ratio = $\frac{V}{V/R} = R$

35.(D) $V_B = iR_{\text{ammeter}} = 0$ $\Rightarrow$ $V_{\text{cap.}} = 0$

36.(A) **37.(A)** **38.(C)**



$-E_2 + E_1 - iR_1 = 0$

$E_2 - (i - x)R_2 - E_2 = 0$

$i = x \quad ; \quad i = \dfrac{E_1 - E_2}{R_1}$

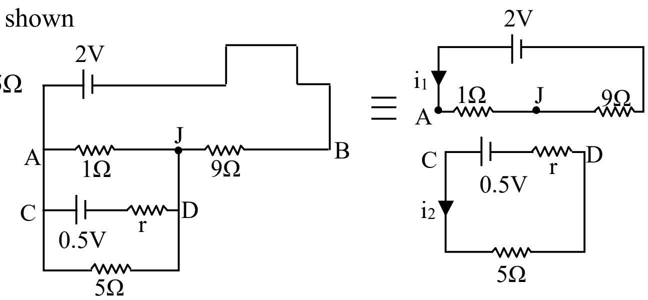
$V_A - E_1 + E_2 = V_B \Rightarrow V_A - V_B = E_1 - E_2 = \text{constant}.$

After long time, $V_C = \varepsilon_1 - I R_1$ (current through capacitor = 0) $\Rightarrow \varepsilon_1 - \left(\dfrac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2}\right) R_1 = \dfrac{\varepsilon_1 R_2 + \varepsilon_2 R_1}{R_1 + R_2}$

Hence $Q_C = V_C \cdot C = C\left(\dfrac{\varepsilon_1 R_2 + \varepsilon_2 R_1}{R_1 + R_2}\right)$

39.(A) Equivalent diagram when both k_1 and k_2 are closed is as shown

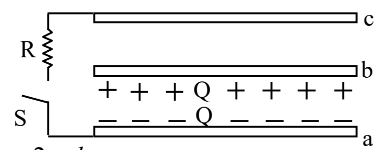
$\Rightarrow \quad V_{AJ} = V_{CD} \Rightarrow \dfrac{2 \times 1}{10} = \left(\dfrac{0.5}{5 + r}\right) \times 5 \Rightarrow r = 7.5\Omega$



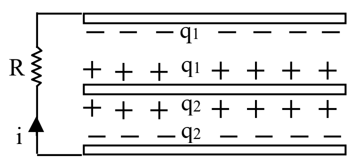
40.(A) $E_2 = E_1 \cdot \dfrac{\ell_{AJ}}{\ell_{AB}} \Rightarrow \ell_{AJ} = \dfrac{E_2 \ell_{AB}}{E_1} = 12.5\ cm$

41.(C) Before closing switches

Energy stored $= \dfrac{Q^2}{2(A \in_0 / d)} = \dfrac{60 \times 60}{2 \times 6} \mu J = 300\ \mu J$



42.(A) At time t



KVL : $\dfrac{q_2 d}{A \in_0} - \dfrac{2q_1 d}{A \in_0} + iR = 0$ . . . .**(i)**

$q_1 + q_2 = Q$ . . . .**(ii)**

$i = -\dfrac{dq_1}{dt}$ . . . .**(iii)**

$\Rightarrow \quad \dfrac{Q - q_1}{C} - \dfrac{2q_1}{C} - R\dfrac{dq_1}{dt} = 0$

$\left(\dfrac{A \in_0}{d} = C\right) \qquad \Rightarrow \quad -R\dfrac{dq_1}{dt} = \dfrac{3q_1 - Q}{C}$

$\Rightarrow \quad -R\displaystyle\int_0^{q_1} \dfrac{dq_1}{3q_1 - q} = \dfrac{1}{C}\int_0^t dt \quad \Rightarrow \quad \dfrac{-R}{3}\ell n\left(\dfrac{3q_1 - Q}{-Q}\right) = \dfrac{t}{C} \quad \Rightarrow \quad \dfrac{-3q_1}{Q} + 1 = e^{-3t/RC}$

$\Rightarrow \quad q_1 = \dfrac{Q}{3}\left(1 - e^{-3t/RC}\right) = 20\left(1 - e^{-500t}\right) \mu C$

43.(D) $q_2 = 60 - 20 = 40 \mu C \ at\ t = \infty$

44. **[A-qs ; B-p ; C-p ; D - t]**

From charge conservation, current through any cross section remains constant. Current density is current/Area of cross section. Only in option A cross sectional area is constant. Hence Q fits only with A.

Similarly, from $i = n_0eAV_d$, drift velocity depends on the ratio of current and cross sectional area. So, the above argument applies in part (S) as well.

45. **[A - p r s ; B – p r s ; C – r ; D - q r]**

		Q	**C**	**V**
(A)	a	C_0V_0	C_0	V_0
	d	$\frac{C_0V_0}{2}$	$\frac{C_0}{2}$	V_0
	c	$\frac{C_0V_0}{2}$	$\frac{C_0}{2}$	V_0
	b	$\frac{C_0V_0}{2}$	$\frac{KC_0}{K+1}$	$\frac{V_0(K+1)}{2K}$
(B)	d	0	$\frac{C_0}{2}$	0
	a	$V_0\frac{C_0}{2}$	$\frac{C_0}{2}$	V_0
	c	$V_0\frac{C_0}{2}$	$\frac{C_0}{2}$	V_0
	b	$\frac{C_0V_0}{2}$	$\frac{KC_0}{K+1}$	$\frac{V_0(K+1)}{2K}$
(C)	b	0	$\frac{KA\varepsilon_0}{d}$	0
	a	KC_0V_0	KC_0	V_0
	c	KC_0V_0	KC_0	V_0
	d	KC_0V_0	$\frac{KC_0}{K+1}$	$(K+1)V_0$
(D)	a	C_0V_0	C_0	V_0
	b	KC_0V_0	KC_0	V_0
	d	$\frac{KC_0V_0}{K+1}$	$\frac{KC_0}{K+1}$	V_0
	c	$\frac{KC_0V_0}{K+1}$	$\frac{KC_0}{K+1}$	V_0

46. **[A – p q ; B – s ; C – r ; D – q]**

It is easy to see that the capacitor will initially drive the current in the same direction as battery (Anti-clockwise). The capacitor will discharge after some time. Then it will get charged in the opposite direction, the direction of current remaining anti-clockwise throughout.

Graph between charge and current can be drawn from the KVL equation around the circuit.

47. **[A-p, q]** **[B-q, s]** **[C-q, s]** **[D-q]**

48.(1) Assuming potential at $B = 0$

$I_2 = \frac{30-5}{25} = 1$; $I_1 = \frac{30-25}{5} = 1$

49.(5) The equivalent resistance at 0°C is

$$R_0 = \frac{R_{10}R_{20}}{R_{10}+R_{20}} \quad \textbf{... (i)}$$

The equivalent resistance at t°C is

$$R = \frac{R_1R_2}{R_1+R_2} \quad \text{... (ii)}$$

But $R_1 = R_{10}(1+\alpha t)$ ... (iii)

$R_2 = R_{20}(1+2\alpha t)$ ... (iv)

And $R = R_0(1+\alpha_{eff}t)$ ... (v)

Putting the value of (i), (iii), (iv), (v) in eqn. (ii), ; $\alpha_{eff} = \frac{5}{4}\alpha$

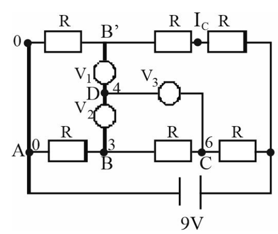
50.(2) Let $x \rightarrow$ be the number of electrons striking the surface per unit time.

$F = PA = nmv = \frac{I}{q}mv$

$I = \frac{PAq}{mv}$; $I = \frac{9.1\times1\times10^{-4}\times1.6\times10^{-19}}{9.1\times10^{-31}\times8\times10^{7}} = 2$ Ampere

51.(1) Current in circuit

$I = \frac{1}{5}A = 0.2$ A which will pass through 10 Ω and 20Ω in both the cases.



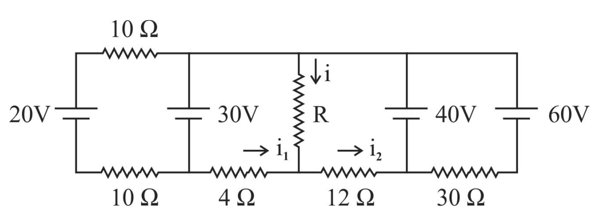
52.(2) Taking potential at A to be zero, potential at B = 3V and potential at B' = 3V and potential at C = 6V

Let V_D be potential of point D then sum of charges reaching point D is zero

$$\frac{V_B - V_D}{R_{V_2}} + \frac{V_{B'} - V_D}{R_{V_1}} + \frac{(V_C - V_D)}{R_{V_3}} = 0 \qquad \left[R_{V_1} = R_{V_1} = R_{V_3} = R\right]$$

$\Rightarrow \quad \frac{3-V_D}{R} + \frac{3-V_D}{R} + \frac{6-V_D}{R} = 0 \quad \Rightarrow \quad 12 - 3V_D = 0$; $V_D = 4$ volts $\Rightarrow$ reading of $V_3 = 2$ volts.

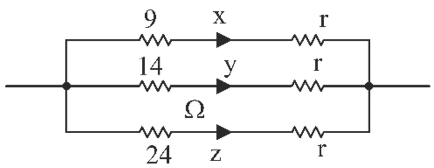
53.(3)



$R_{eq} = R_{in} = \frac{4\times12}{4+12} = 3\Omega$ (Max power delivered when internal resistance = External resistance)

54.(3) **(1)** $x(9+r) = y(14+r)$

(2) $xr = 7.5$



(3) $yr = 5$

By (1), (2), (3)

$9x + 7.5 = 14y + 5 = 24z + zr$

So, $9x - 14y = -2.5$; $14y - 9x = 2.5$

So, $24z + zr = 9\left(\frac{7.5}{r}\right) + 7.5$; $z = \frac{7.5\left(\frac{9}{r}+1\right)}{24+r} = \frac{7.5(9+r)}{r(24+r)}$

So, $14\left(\frac{5}{r}\right) + 5 = \frac{24 \times 7.5 \times (9+r)}{r(24+r)} + \frac{7.5(9+r)}{(24+r)}$

$= \frac{(9+r)(7.5)}{(24+r)}\left[\frac{24}{r}+1\right]$; $5\left(\frac{14}{r}+1\right) = \frac{(9+r)(7.5)}{r} \Rightarrow 5(14+r) = 7.5(9+r)$

$\Rightarrow$ $70 + 5r = 67.5 + 7.5r \Rightarrow 2.5 = 2.5r \Rightarrow r = 1$

$\Rightarrow$ $x = 7.5 \Rightarrow y = 5 \Rightarrow 24z + z = 14(5) + 5$; $25z = 75 \Rightarrow z = 3$

Voltage across $V_3 = zr = 3$

55.(0) Since potential difference across AD and across CD is same, so A & C are on same potential. Therefore energy stored in the capacitor at given instant is zero.

56.(4) Current through branch containing capacitor is

$\frac{dq}{dt} = I = 3e^{-t}$ amp. At $t = 0, I = 3A$

In vertical branch

$20 - V_P = 3 \times 1$; $V_P = 17$

In left branch

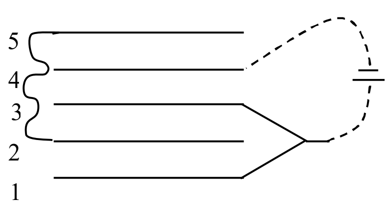
$18 - V_P = I' \times 1$

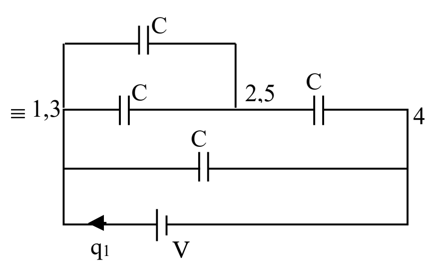
$18 - 17 = I' \times 1$

Current through resistance R is $I + I' = 3 + 1$

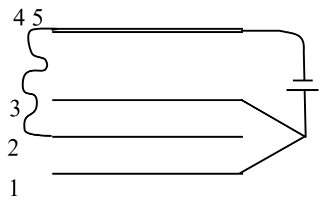
4 amp.

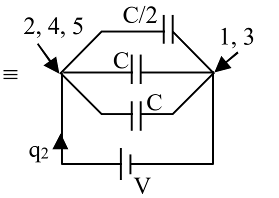
57.(2.5)





$\Rightarrow$ $q_1 = \frac{5CV}{3}$

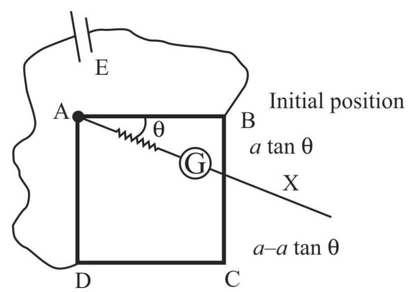
Again 

$\equiv$ 

$\Rightarrow \quad q_2 = \dfrac{5CV}{2}$

$\Rightarrow$ Work done by battery $= V(q_2 - q_1) = CV^2\left(\dfrac{5}{2} - \dfrac{5}{3}\right) = \dfrac{5CV^2}{6} = \dfrac{5}{6} \times 30 \times (10)^2\ \mu J = 2.5 mJ$

58.(2) Let a be the side length of square and θ be the position where galvanometer gives zero deflection. To have zero deflection bridge is to be balanced.



$$\Rightarrow \quad \frac{R_{AB}}{R_{AD}} = \frac{R_{BX}}{R_{DC} + R_{CX}}$$

[R_{DC} and R_{CX} is in series]

$$\frac{100}{200} = \frac{\frac{400}{a} a \tan\theta}{500 + \frac{400}{a}(a - a\tan\theta)}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{400\tan\theta}{500 + 400(1 - \tan\theta)}$$

Solving $\tan\theta = 3/4$,

t be the time taken from start, $\theta = \omega t [\theta \text{ is radian}]$

$$\frac{\pi}{180} \times 37 = \frac{\pi}{360} \times t \qquad \Rightarrow \qquad t = 74 \text{ sec} = 2 \times 37 \text{ sec.}$$

59.(1.4) Use result of Req. for cross symmetry

$$\frac{1}{\text{Req}} = \frac{\frac{2}{R_1R_2} + \frac{1}{R_2R_3} + \frac{1}{R_3R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{2}{R_3}} \qquad R_1 = R,\ R_2 = 2R,\ R_3 = R$$

$$\frac{1}{\text{Req}} = \frac{\frac{2}{2R^2} + \frac{2}{2R^2} + \frac{1}{R^2}}{\frac{7}{2R}} = \frac{\frac{1}{R^2}\left(\frac{5}{2}\right)}{\frac{7}{2R}} = \frac{5}{2R^2} \cdot \frac{2R}{7} = \frac{5}{7R} \qquad \text{Req.} = \frac{7R}{5}$$

60.(3) During charging for $\tau_1 = \text{Req } C$

Req $R_3 = R$

$\tau_1 = RC$

During discharge $= \tau_2 = \text{Req } C$

$$\text{Req} = \frac{R_1R_2}{R_1 + R_2} + R_3 = \frac{3R}{2}$$

$$\tau_2 = \frac{3}{2}RC$$

$$\frac{\tau_1}{\tau_2} = \frac{2}{3} = \frac{2}{n} \Rightarrow n = 3$$

MAGNETIC EFFECT OF CURRENT

1.(C) $mv = i\left|\vec{l} \times \vec{B}\right|t = Bilt \Rightarrow v = \dfrac{Bilt}{M} = \dfrac{Blq}{M} \Rightarrow \quad 0 = v^2 - 2gh \Rightarrow \dfrac{B^2l^2q^2}{M^2} = 2gh$

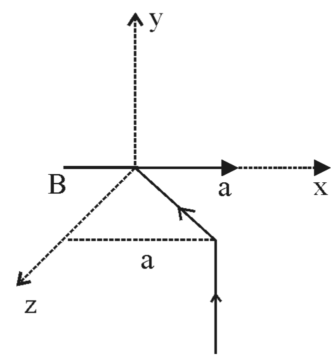
2.(B) $B(x) = \dfrac{\mu_0 I_2}{2\pi x},\ dF = BI_1dx = \dfrac{\mu_0 I_1 I_2 dx}{2\pi x} \Rightarrow \quad F = \dfrac{\mu_0 I_1 I_2}{2\pi}\int_a^{2a}\dfrac{dx}{x} = \dfrac{\mu_0 I_1 I_2}{2\pi}\ln 2$

3.(A) $R = \dfrac{mv}{qB},\ 2R < r \Rightarrow V < \dfrac{qBR}{M} \Rightarrow V < \dfrac{qBr}{2M} = \dfrac{qr}{2M}\mu_0 ni$

4.(C) The plane of motion of the particle is the z-x plane. It is a case of uniform circular motion $\Rightarrow \vec{a}.\vec{v} = 0$

5.(C) Magnetic moment M = $\dfrac{qL}{2m} = \dfrac{q}{2m}\dfrac{2}{5}mR^2\omega \Rightarrow M = \dfrac{1}{5}qR^2\omega$

6.(C) $\vec{B} = \dfrac{\mu_0 I}{4\pi a\sqrt{2}}\left[\dfrac{-\hat{i}}{\sqrt{2}} + \dfrac{\hat{k}}{\sqrt{2}}\right]$



7.(B) $\vec{B} = \dfrac{\mu_0 I_1}{2\pi \times 10^{-2}}\hat{k} + \dfrac{\mu_0 I_2}{2\pi \times 2 \times 10^{-2}}\hat{j}$

8.(D) $T = \dfrac{2\pi m}{qB} = \dfrac{2\pi \times 1}{1 \times 1} = 2\pi s$; $\quad R = \dfrac{mv}{qB} = \dfrac{1\times 1}{1 \times 1} = 1m$

x = 0, $y = \dfrac{1}{2}\dfrac{E_y q}{M}t^2 = \dfrac{1}{2}\dfrac{1\times 1}{1}\times \pi^2 = \dfrac{\pi^2}{2}m$

z = 2 R = 2m

Co-ordinates will be $\left(0, \dfrac{\pi^2}{2}, 2\right)m$

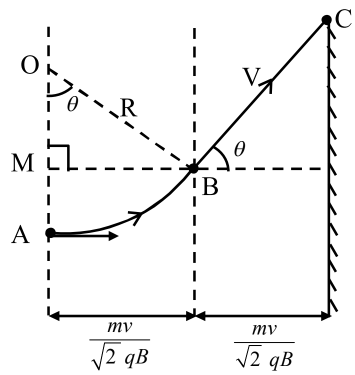
9.(A) Force on $PS = F = I\left(\sqrt{2}\ R\right)B$ Force on $PQR = F' = I(2\ R)B \Rightarrow F' = \sqrt{2}\,F$

10.(A) $R = \dfrac{mv}{qB} > \dfrac{mv}{\sqrt{2}\,qB}$. Hence particle will enter in the region where magnetic field is absent. The return path will be identical to path ABC. So required time $t = 2\left(t_{AB} + t_{BC}\right)$

$= 2\left[\dfrac{\theta m}{qB} + \dfrac{mv/\sqrt{2}\ qB}{cos\,\theta V}\right]$

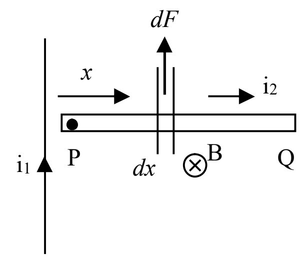
Also $sin\,\theta = \dfrac{mv/\sqrt{2}qB}{mV/qB} = \dfrac{1}{\sqrt{2}}$ from triangle BMO

Hence, $t = 2\left[\dfrac{\pi m}{4qB} + \dfrac{m}{q\,B}\right] = \dfrac{m}{2q\,B}\left[\pi + 4\right]$



11.(B) Speed can change only due to work done by electrostatic force.

Hence $qEZ = \dfrac{1}{2}mv^2 \Rightarrow \quad 10qZ = \dfrac{1}{2}mv^2 \Rightarrow V = \sqrt{\dfrac{20qz}{m}}$



12.(D) $B = \dfrac{\mu_0 I_1}{2\pi}$, let the current in PQ be $i_2 \Rightarrow \quad dF = Bi_2 dx = \dfrac{\mu_0 i_1 i_2}{2\pi x} dx$

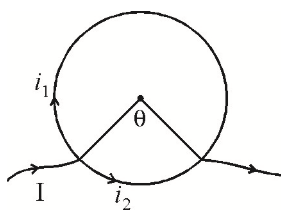
$\Rightarrow$ net anticlockwise torque on PQ is $= \int dF \,.\, x = \int_0^{\ell} \dfrac{\mu_0 i_1 i_2}{2\pi} dx = \dfrac{\mu_0 i_1 i_2 \ell}{2\pi}$

For equilibrium $\dfrac{\mu_0 i_1 i_2 \ell}{2\pi} = mg\dfrac{\ell}{2} \quad \Rightarrow \quad i_2 = \dfrac{\pi mg}{\mu_0 i_1}$

13.(C) $R = \dfrac{mV}{qB}$ is decreasing as v is decreasing

$\Rightarrow$ it enters at P $\Rightarrow$ charge is negative $\because$ bending shows the direction of force

14.(B)



$i_1 = \dfrac{I\theta}{2\pi}\ ;\ i_2 = \dfrac{I(2\pi - \theta)}{2\pi}$

$B_1 = \dfrac{\mu_0 i_1}{2r}\dfrac{(2r-\theta)}{2\pi}$ and $B_2 = \dfrac{\mu_0 i_2}{2r} \,.\, \dfrac{\theta}{2z}$

B_1 and B_2 are in opposite direction, but have same magnitude $= \dfrac{\mu_0 I\theta(2\pi - \theta)}{8\pi^2 r} \quad \Rightarrow$ net field is zero.

15.(A) $B_1 = \dfrac{KI}{a},\ B_2 = \dfrac{KI}{4a\sqrt{2}} - \dfrac{KI/a}{4} = \dfrac{(\sqrt{2}-1)}{4}\dfrac{KI}{a}$

$B_3 = \dfrac{KI}{a/\sqrt{2}} = \dfrac{\sqrt{2}KI}{a} \Rightarrow B_3 > B_1 > B_2$

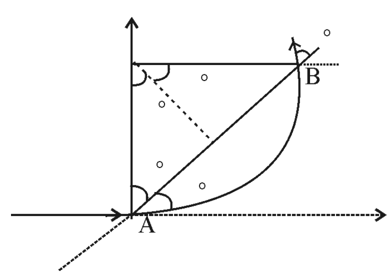
16.(BCD) $\vec{\tau} = \vec{m} \times \vec{B} \quad \Rightarrow \quad \vec{U} = -\vec{m} \,.\, \vec{B}$

17.(BD) Electric field along the axis is non zero due to P.D. along axis.

18.(BCD) Force on *ab* will be stronger than *bc*.

19.(ACD) $K.E = qV = \dfrac{1}{2}mV^2 \quad \Rightarrow \quad R = \dfrac{mV}{qB}$

20.(AD) Use symmetry and Ampere's law

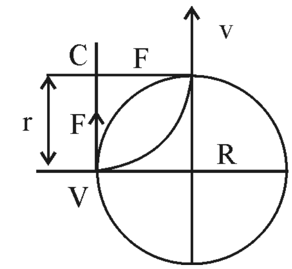


21.(AD) $d_B = \dfrac{\mu_0 \sigma 2\pi x dx}{R \times 2x}\dfrac{d\omega}{2\pi}$; $\quad B = \dfrac{1}{2}\int \mu_0 \omega \dfrac{q}{\pi R^2} dx$

22.(ABC) $Arc AB = \dfrac{\pi}{3} r = \dfrac{\pi m V}{3qB}$

$Time\ 't' = \left(\dfrac{I}{2\pi}\right)\left(\dfrac{\pi}{3}\right) = \dfrac{T}{6} = \dfrac{\pi m}{3qB}$

23.(ABC) The particle will move along an arc which is part of a circle of radius $r = \frac{m\text{v}}{Bq}$



From the figure we can see r = R

$\therefore\ R = \frac{m\text{v}}{Bq};\ T = \frac{\pi r/2}{V} = \frac{\pi r}{2v} \qquad \because\ r = R = \frac{mV}{Bq}\ \therefore T = \frac{\pi m}{2Bq}$

24.(CD) For cylinder $B = \frac{\mu_0 ir}{2\pi R^2};\ r < a\ = \frac{\mu_0 i}{2\pi r};\ r \geq a$

We can consider the given cylinder as a combination of two cylinders. One of radius 'R' carrying current I in one direction and other of radius R/2 carrying current I/3 in both directions.

At point A: $B = \frac{\mu_0 (I/3)}{2\pi (R/2)} + 0 = \frac{\mu_0 I}{3\pi R}$ At point B: $B = \frac{\mu_0 (4I/3)}{2(\pi R^2)}\left(\frac{R}{2}\right) + 0 = \frac{\mu_0 I}{3\pi R}$

25.(ABC) y is speed of light x and z also have same dimension

26.(ABC) $\vec{F} = I\vec{l} \times \vec{B}$.

27.(AD) Normal force of the rail on the wire = Bil

$\Rightarrow$ max force of friction at $t = 0$ is $Bi_{\text{intial}}\ l.\ \mu\ = 2 \times \frac{100}{5}\ .\ 1\ .\frac{3}{4} = 30N$

But weight $= 2g\ = 20N$ $\Rightarrow$ force of static friction at $t = 0$ is 20 N

Normal force at time t is Bil $= 2 \times \frac{100}{5 + 0.5t}\ .\ 1$ $\Rightarrow$ normal force is decreasing

$\Rightarrow$ friction is also decreasing max. value of force static

$\Rightarrow$ When max. frictional force reduce to weight of the rod, it stats moving $= \frac{200}{5 + 0.50t} \times \frac{3}{4} = 20$

$\Rightarrow$ $30 = 20 + 2t\ \Rightarrow\ t = 5sec$

28.(BC) Consider the solid cylinder as super position of solenoids.

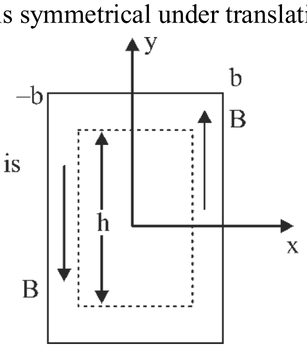
29.(ABC) $R = \frac{L}{A}$, if we double radius and cross-sectional radius, then resistance will be halved.

30.(ACD) Due to sheet

$B = \frac{\mu_0 k}{2} = \frac{\mu_0 (2bJ)}{2} = \mu_0 JB.$

The slab is symmetrical under translation in y so field is independent of y.

Also slab is



Symmetric under rotation by 180° around Z axis, so y component of field is odd function of x. consider the ampere loop shown in diagram

$$2Bh = \mu_0(2xhJ) \qquad \therefore \qquad B = \mu_0 Jx$$

31.(C)

32.(ABCD)

(A) We have $\vec{F} = q\left[\vec{E} + (\vec{V} \times \vec{B})\right]$

$$F = q\left[a\hat{i} + (x\hat{i} + y\hat{i}) \times b\hat{i} + c\hat{i} + d\hat{k}\right]$$

$$\vec{F} = q\left[\hat{i}(a + yd) - \hat{j}(xd) + \hat{k}(xc - yb)\right]$$

(B) for $c = 0; d = 0; y = 0$

$$F = qa\hat{i}$$

So particle will move along straight line with increasing speed

(C) for $c = 0; d = 0; y = 0$

$$F = q[(a + yd)i - yb\,\hat{k}]$$

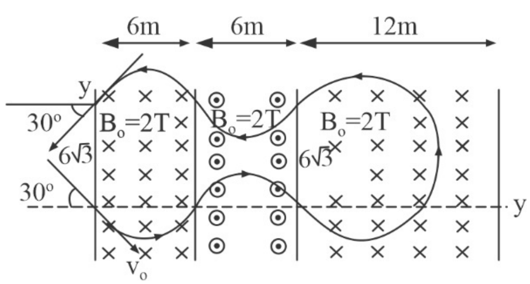
$$|F| = q\sqrt{(a + yb)^2 + (yb)^2}$$

So particle will moves along helix of varying pitch

(D) Here $a = 0, \vec{v} \cdot \vec{B} = 0$

And v is perpendicular to B so particle will move in circular both.

33.(AB)



34.(AC) $T = \dfrac{2\pi m}{Bq} = \dfrac{2\pi}{\alpha B_0}$

At $t = \dfrac{\pi}{\alpha B_0} = \dfrac{T}{2}$; velocity of particle is $-v_0\hat{i} + v_0\hat{k}$

Speed will always remains constant $= v_0\sqrt{2}$

At $t = \dfrac{2\pi}{\alpha B_0} = T$; displacement is equal to pitch, $\Delta x = V_0 T = \dfrac{2\pi V_0}{\alpha B_0}$

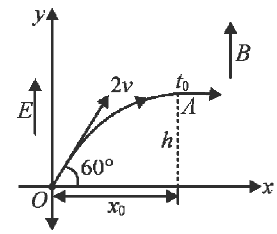
At $t = \dfrac{2\pi}{\alpha B_0} = T$; distance = speed $\times T = \dfrac{2\sqrt{2}V_0\pi}{\alpha B_0}$

35.(CD) Voltage sensitivity $= \dfrac{NBA}{R} = \dfrac{NBA}{\frac{N\rho\ell}{A'}} = \dfrac{BAA'}{\rho\ell}$

Independent of number of coils

By changing the dimensions area many remain same.

36.(A) First particle will travel along parabolic path OA. Let time from O to A is t. $a_y = \frac{-qE}{m}$

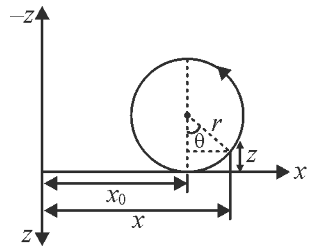


$$x = \frac{\sqrt{3}mv^2}{qE} = (2v\cos 60°)t_0 \quad \Rightarrow \quad t_0 = \frac{\sqrt{3}mv}{qE}$$

$$v_y = u_y + a_y t_0 = 2v\sin 60°\ ,\ -\frac{qE}{m}\frac{\sqrt{3}mv}{qE} = 0$$

Hence at point A, velocity will be purely along x-axis and it will be $2v\cos 60° = v$.

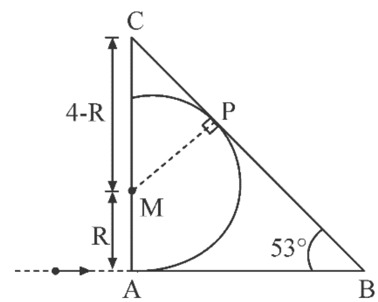
37.(B) Now magnetic field is switched on along y-axis. Now its path will be helical as shown below with increasing pitch towards negative y-axis. $r = \frac{mv}{qB}$



$$x = x_0 + r\sin\theta = (2v\cos 60°)t_0 + \frac{mv}{qB}\sin\omega t \qquad = v\sqrt{3}\frac{mv}{qE} + \frac{mv}{qB}sin\left(\frac{qB}{m}t\right)$$

38.(C) z-coordinate : $z = -(r - r\cos\theta) \qquad = -\frac{mv}{qB}\left[1-\cos\left(\frac{qB}{m}t\right)\right]$

39.(C) In triangle PMC



$$\cos 53° = \frac{MP}{MC}$$

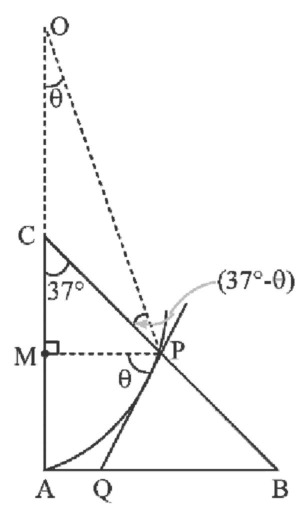
$$\frac{3}{5} = \frac{R}{4-R}$$

12 = 8R

$R = \frac{3}{2}m$ (R is the maximum radius of half – circle)

$$R_{max} = \frac{mu_{max}}{qB} \Rightarrow u_{max} = 3\ m/s$$

40.(B) $R = \frac{mu}{qB} = 24\,m$



Let, $\angle MPQ = \theta$

By geometry

$\angle CPO = (37 - \theta)$

In ΔCPO

$$\frac{OC}{\sin(\angle CPO)} = \frac{OP}{\sin(\angle PCO)}$$

$$\frac{20}{\sin(37° - \theta)} = \frac{24}{\sin(180° - 37°)}$$

$$\Rightarrow \frac{5}{\sin(37^\circ - \theta)} = \frac{5\times 6}{3} \Rightarrow \sin(37^\circ - \theta) = \frac{1}{2}$$

$$\theta = \frac{7\pi}{180} rad. \Rightarrow \omega = \frac{qB}{m} \Rightarrow \omega = 2 rad/\sec. \Rightarrow t = \frac{7\pi}{360} \sec.$$

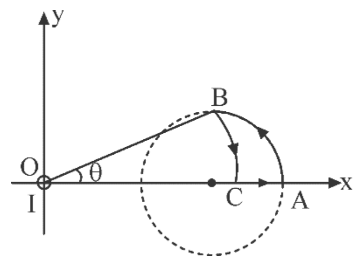
41.(D) Since there is no current passing through circular path, the integral $\oint \vec{B}\cdot d\vec{\ell}$ along the dotted circle is zero.

42.(B) Let segment OB = OC and arc BC is a circular arc with centre at origin. Since the shown closed path ABCA encloses no current, the path integral of magnetic field over this path is zero.

Hence $\int_A^B \vec{B}\cdot\overrightarrow{d\ell} + \int_B^C \vec{B}\cdot\overrightarrow{d\ell} + \int_C^A \vec{B}\cdot\overrightarrow{d\ell} = 0$

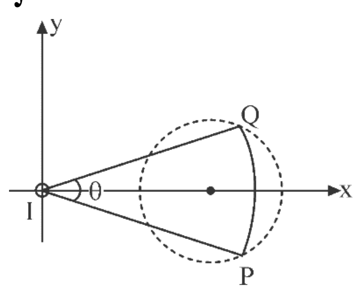
Because $\vec{B}$ is perpendicular to segment AC at all points,

therefore $\int_C^A \vec{B}\cdot\overrightarrow{d\ell} = 0$



Hence $\int_A^B \vec{B}\cdot\overrightarrow{d\ell} = \int_C^B \vec{B}\cdot\overrightarrow{d\ell} = \frac{\mu_0 I}{2\pi}\frac{OB(\theta)}{2\pi} = \frac{\mu_0 I}{2\pi}\tan^{-1}\frac{1}{2}$

43.(C) Consider two points P and Q lying on dotted circle and equidistant from origin O. We draw a circular arc QP with centre origin O. The path integral of magnetic field, that is, $\int \vec{B}\,.d\vec{\ell}$ along the dotted circle between two points P and Q is also is equal to path integral $\int \vec{B}\cdot d\vec{\ell}$ along the arc QP whose cenre is at origin.



Therefore the path integral of magnetic field $\int \vec{B}\cdot d\vec{\ell}$ along the dotted circle between two points P and Q

$$= \frac{\mu_0 I}{2\pi}\frac{OP(\theta)}{OP} = \frac{\mu_0 I}{2\pi}\theta$$

The value of θ will be maximum when chord OQ and chord OP will be tangent to the dotted circle, that is, $\theta = \frac{\pi}{3}$.

Hence the required maximum value $= \frac{\mu_0 I}{6}$.

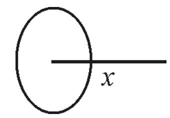
44. **[A – s ; B - p ; C – q ; D – r]** Use $\vec{F} = q\left(\vec{V}\times\vec{B}\right) \Rightarrow R = \frac{mV}{qB}$

45. **A-p, q; B-p, s; C-p, r; D-p**

46.(9) Energy density of electric field, $u_\tau = \frac{1}{2} \in_0 E^2$ Energy density of magnetic field. $u_B = \frac{1}{2}\frac{B^2}{\mu_0}$

47.(5) $B_z = -\frac{\mu_0 I}{4\pi R} = -10^{-7} \times \frac{10}{10 \times 10^{-2}} = -10^{-5} T$

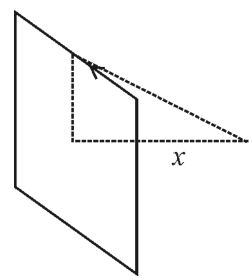
48.(3) $B_p = \frac{\mu_0 I}{4\pi} \frac{2\pi R^2}{\left(R^2 + x^2\right)^{3/2}} = \frac{\mu_0 I}{2} \frac{R^2}{\left(R^2 + x^2\right)^{3/2}}$



$B_c = \frac{\mu_0 I}{2R}$ $\Rightarrow \frac{\mu_0 I}{16R} = \frac{\mu_0 I}{16R} \frac{R^2}{\left[R^2 + x^2\right]^{3/2}}$ $\Rightarrow 8R^3 = \left(\left[R^2 + x^2\right]^{1/2}\right)^3$

$2R = \sqrt{R^2 + x^2}$ $\Rightarrow 4R^2 = R^2 + x^2$ $\Rightarrow R = \sqrt{3}x$ $\Rightarrow k = 3$

49.(4) $B_1 = \frac{\mu_0 I}{4\pi\sqrt{x^2 + \frac{l^2}{4}}}$, $B = 4 \times \frac{\mu_0 I}{4\pi\sqrt{x^2 + \frac{l^2}{4}}} \left[\frac{l}{2\sqrt{x^2 + \frac{l^2}{4}}}\right]$



50.(5) $F_m = qvB$, and directed radially outward.

$\because N - mg\sin\theta - qvB = \frac{mv^2}{R}$ (N will be max at $\theta = 90°$)

$\Rightarrow N_{\max} = \frac{2mgR}{R} + mg + qB\sqrt{2gR} = 3mg + qB\sqrt{2gR}$

51.(7) The magnetic field due to ring in x-y plane is

$\vec{B}_1 = \frac{\mu_0 I}{2R}\hat{k}$

the magnetic field due to ring in y-z plane is

$\vec{B}_2 = \frac{\mu_0 I}{2R}\hat{i}$

and the magnetic field due to ring in x-z plane is

$\vec{B}_3 = \frac{\mu_0 I}{2R}\hat{j}$ $\therefore$ $\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$

$\vec{B} = \frac{\mu_0 i}{2R}\left(\hat{k} + \hat{i} + \hat{j}\right)$ $\therefore$ $B = \frac{\sqrt{3}\mu_0 I}{2R} = \sqrt{\frac{3}{4}}\left(\frac{\mu_0 I}{R}\right)$

Hence, X = 3 and Y = 4

52.(2) To enter region 2; Radius in region I should be greater than $`d'$

$R = \frac{mV}{qB} > d$; $V > \frac{qBd}{m}$; $V > \frac{1.6 \times 10^{-19} \times 0.001 \times 5 \times 10^{-2}}{9 \times 10^{-31}}$; $V > \frac{8}{9} \times 10^7$

To come out of region 2; diameter $2R > d$

$\frac{2mV}{qB} > d$; $V > \frac{qBd}{2m}$; $V > \frac{1.6 \times 10^{-19} \times 0.002 \times 5 \times 10^{-12}}{2 \times 9 \times 10^{-31}}$

$V > \frac{8}{9} \times 10^7 \text{ ms}^{-1}$

Hence for both region $V > \frac{8}{9} \times 10^7 \text{ ms}^{-1}$

53.(3) The magnitude o f magnetic moment is

$M = iA = 10 \times (10 \times 10^{-2})^2 Am^2 = 10 \times 10^{-2} = 0.1 Am^2$

The normal on the loop is in $x - z$ plane. It makes 60° angle with x-axis.

$\therefore \quad \vec{M} = M \cos 60° \hat{i} - M \sin 60° \hat{j} \qquad \therefore \quad \vec{M} = \frac{M}{2}\hat{i} - \frac{\sqrt{3}}{2}M\hat{j}$

$\therefore \quad \vec{M} = \frac{0.1}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right) \quad ; \quad \vec{M} = (0.05)\left(\hat{i} - \sqrt{3}\,\hat{j}\right)\text{Am}^2 \;;\; X = 3$

54.(4) Balancing torque

$$mg \times \frac{2R}{\pi} = I\frac{\pi R^2}{2}B$$

$$\frac{\pi}{10} \times (\pi R) \times g \times \frac{2R}{\pi} = I\frac{\pi R^2}{2}B \quad : \quad \frac{4}{B} = I$$

$I = 4$

55.(25) Current sensitivity,

$$I_s = \frac{\alpha}{I}$$

Voltage sensitivity,

$$V_5 = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{I_S}{R}$$

New curent sensitivity,

$$I_s = I_s + \frac{50}{100}I_s = \frac{3}{2}I_s$$

New voltage sensitivity,

$$V_s' = \frac{I_s'}{2R} = \frac{\frac{3}{2}I_s}{2R} = \frac{3}{4}V_s = 0.75V_s$$

Thus new voltage sensitigvity becomes 75% of its initial value i.e., it decreases by 25%

56.(5) $\oint B \cdot d\vec{\ell} = \mu_0 i_{net}$

$$B \times 2\pi r = \mu_0 \int_0^r \left(J_0 \frac{r}{a}\right) 2\pi r^2 dr$$

57.(7) $J = \dfrac{I}{\pi(b^2 - a^2)}$

$$\oint B \cdot d\ell = \mu_0 I_{enclosed}$$

$B \times (2\pi r) = \mu_0[J \times \pi(r^2 - a^2)]$; $B = \frac{\mu_0}{2\pi} \times \frac{I}{r} \times \left(\frac{r^2 - a^2}{b^2 - a^2}\right)$

$= 2 \times 10^{-7} \times \frac{5.5}{1.1 \times 10^{-2}} \times \frac{(0.21)}{(3)}$; $= 7\mu T$

58.(2) Magnetic field is non zero only in the region between the two solenoids, where $B = \mu_0 n_2 i_2$

$\therefore$ Energy stored per unit volume $= \frac{B^2}{2\mu_0} = \frac{\mu_0 n_2^2 i_2^2}{2}$

The energy per unit length = energy per unit volume × area of cross section where $B \neq 0$

$\frac{\mu_0 n_2^2 i_2^2}{2}[\pi(r_2^2 - r_1^2)] = \frac{\mu_0 n_1^2 i_1^2}{2}[\pi(r_2^2 - r_1^2)]$

(Since $n_1 i_1 = n_2 i_2$)

59.(3) $\vec{E}$ is parallel to $\vec{B}$ and $\vec{v}$ is perpendicular to both. Therefore, path of the particle is a helix with increasing pitch.

Speed of particle at any time t is

$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ **. . . (i)**

Here, $v_y^2 + v_z^2 = v_0^2$ and $v_x^2 = \left(\frac{qE}{m}t\right)^2$ and $v = 2v_0$

Substituting the values in eq. (i), we get

$t = \frac{\sqrt{3}mv_0}{qE}$

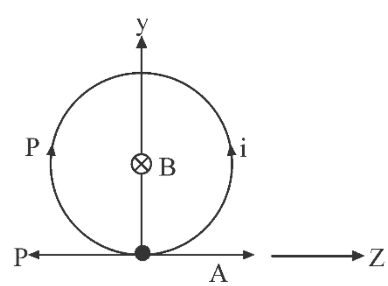
60.(4) A and P will have same momentum in magnitude and they will move in opposite direction. They will move in the circle of same radius and the same centre but in opposite directions. If they meet after time t then

$\omega_A t + \omega_P t = 2\pi$

$\Rightarrow$ $t = \frac{2\pi}{\omega_A + \omega_P} = \frac{2\pi}{\frac{2eB}{4m} + \frac{2eB}{(A-4)m}}$

$t = \frac{4(A-4)m\pi}{eBA}$; $\theta_A = \omega_A t = \frac{2eB}{4m} \times \frac{4m(A-4)\pi}{eBA}$

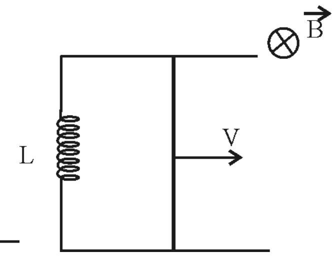
$= \frac{2(A-4)\pi}{A} = \frac{48}{25}\pi \Rightarrow n = 48$



ELECTROMAGNETIC INDUCTION

1.(B) $BVl - L\frac{di}{dt} = 0$; At maximum current $\frac{di}{dt} = 0 \Rightarrow V = 0$

Conservation of energy $\Rightarrow \frac{1}{2}mV_0^2 = \frac{1}{2}LI_m^2 \Rightarrow I_m = V_0\sqrt{\frac{m}{L}}$



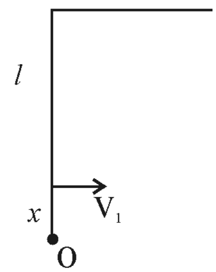
2.(C) Let 'O' be the instantaneous centre of rotation.

$\omega x = V_1$

$\omega(x+l) = V_2 \Rightarrow \frac{V_2}{V_1} = \frac{x+l}{x} \Rightarrow V_2 x = V_1 x + V_1 l$

$\Rightarrow x = \frac{V_1 l}{V_2 - V_1}$; $\omega = \frac{V_1(V_2 - V_1)}{V_1 l} = \frac{V_2 - V_1}{l}$

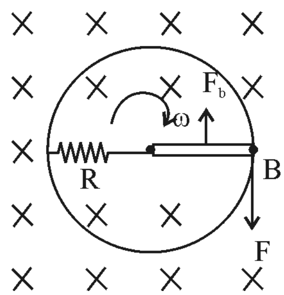
Emf = $\frac{1}{2}B\omega\left[(l+x)^2 - x^2\right]$



3.(D) $F_b = BIL$

Induced current: $I = \frac{(B\omega r^2/2)}{R}$

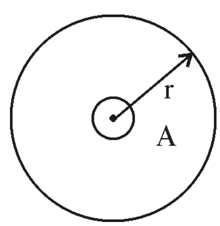
$\therefore F_b = B\left(\frac{B\omega r^2}{2R}\right)r = \frac{B^2\omega r^3}{2R}$



To maintain constant angular velocity: F(r) = $F_B(r/2) \Rightarrow F = \frac{F_B}{2} = \frac{B^2\omega r^3}{4R}$

4.(D) $\frac{d\phi}{dt} = R\frac{dq}{dt} \Rightarrow q = \left|\frac{\Delta\phi}{r}\right|$

5.(A) $\Delta\phi = \frac{\mu_0 i}{2r}An \Rightarrow q = \frac{\Delta\phi}{R} = \frac{\mu_0 nAi}{2rR}$



6.(A) Induced emf $\int_a^b Bvdx = \int_a^b \frac{\mu_0 I}{2\pi x}vdx$

Induced emf = $\frac{\mu_0 Iv}{2\pi}\ln\left(\frac{b}{a}\right) \Rightarrow$ Power dissipated = $\frac{E^2}{R}$

Also, power = F. V $\Rightarrow F = \frac{E^2}{VR} \Rightarrow F = \frac{1}{VR}\left[\frac{\mu_0 IV}{2\pi}\ln\left(\frac{b}{a}\right)\right]^2$

7.(A) W = (L) F = L ×ILB = $L \times \frac{L^2B^2V}{R} = 1J$

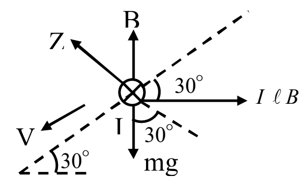
8.(A) The growth of current is given by $i = i_0\left(1 - e^{-t/\tau}\right) \Rightarrow \frac{di}{dt} = \frac{i_0}{\tau}e^{-t/\tau}$

Energy stored in the form of magnetic field energy is: $U_B = \frac{1}{2}Li^2$

Rate of increase of magnetic field energy is: $R = \frac{dU_B}{dt} = Li\frac{di}{dt}$

This will be maximum when $\frac{dR}{dt} = 0 \quad \Rightarrow \quad e^{-t/\tau} = 1/2$ Substituting: $R_{\max} = \frac{Li_0^2}{\tau}$

9.(B) For constant speed $I\ell B\ cos\ 30 = m\,g\ sin\ 30$



$\Rightarrow \quad \left(\frac{B\ell V\ cos\ 30^\circ}{R}\right)\ell B\ cos\ 30 = mg\ sin 30\ \left(\varepsilon_{ind} = B\ell v\ cos\ 30^\circ\right)$ (Side view)

$\Rightarrow \quad B = \sqrt{\frac{2mg\,R}{3v\ell^2}}$

10.(B) When K_1 and K_3 are closed charge on capacitor and current in inductor is given by

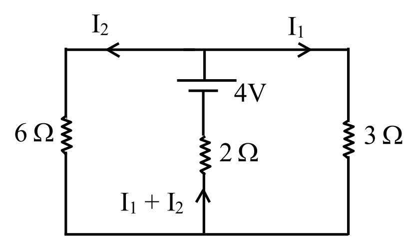
$$q = c\varepsilon\left(1 - e^{-t/RC}\right) = 2\times 2\left(1 - e^{-t/0.5\times 2}\right) = 4\left(1 - e^{-t}\right) = 4\left(1 - \frac{1}{e}\right)\ at\ t = 1 sec$$

$$I = \frac{\varepsilon}{R}\left(1 - e^{-\frac{Rt}{L}}\right) = \frac{4}{2}\left(1 - e^{-\frac{2t}{2}}\right) = 2\left(1 - e^{-t}\right) = 2\left(1 - \frac{1}{e}\right) at\ t = 1\ sec.$$

Now when K_1, K_3 are opened and K_2 is closed then it is an $L - C\ ckt$ and Let q_0 be maximum charge.
From conservation of energy for L-$C\ ckt$

$$\frac{q_0^2}{2C} = \left(\frac{q^2}{2C} + \frac{1}{2}L\,I^2\right) \Rightarrow \frac{q_0^2}{2\times 2} = \frac{4^2\left(1 - \frac{1}{e}\right)^2}{2\times 2} + \frac{1}{2}\times 2\times 2^2\left(1 - \frac{1}{C}\right)^2 \Rightarrow q_0 = 4\sqrt{2}\left(1 - \frac{1}{e}\right)$$

11.(C) $\varepsilon_{ind} = B\ell V = 2\times 1\times 2 = 4\ volt.$

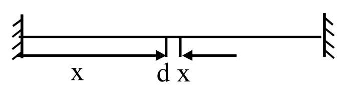


$4 - 3I_1 - 2\left(I_1 + I_2\right) = 0$

$4 - 6I_2 - 2\left(I_1 + I_2\right) = 0$

Solving $I_1 = \frac{2}{3}A, I_2 = \frac{1}{3}A$; $F = \left(I_1 + I_2\right)\ell B = \left(\frac{2}{3} + \frac{1}{3}\right)\times 1\times 2 = 2N$

12.(C) ε_{ind} is maximum when velocity is maximum.



$$\varepsilon_{max} = \int B\,d\,x\,V_{max} = \int_0^L B\,d\,x\left(A\,\omega\sin K\,x\right)$$

$$= -BA\omega\,.\left[\frac{cos\ K\,x}{K}\right]_0^L = \frac{2\,L\,B\,A\omega}{\pi}\ \left(K = \frac{2\pi}{\lambda}, \frac{\lambda}{2} = L\right)$$

13.(C) Let it takes time Δt to switch off the magnetic field

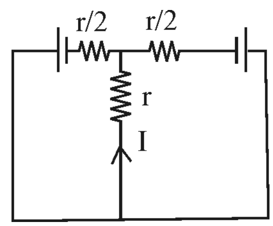
$\Rightarrow$ Included emf $= \frac{B\pi R^2}{\Delta t}$ $\Rightarrow$ Field produced $= \frac{B\pi R^2}{\Delta T} \div 2\pi R = \frac{BR}{2\Delta t}$

$\Rightarrow$ Force experienced $= \frac{BRq}{2\Delta t}$ $\Rightarrow$ velocity gain $= \int_0^{\Delta t} adt = \frac{BRq}{2m} = \sqrt{5gR} \Rightarrow \frac{q}{m} = \frac{2}{B}\sqrt{\frac{5g}{R}}$

14.(ABD) Rate of work done by external agent is:

$$\frac{dw}{dt}=\frac{BIL.dx}{dt}=BILv$$ and thermal power dissipated in the resistor = eI = (BvL) I

15.(BD) Equivalent circuit is $E = B\omega\frac{a^2}{2}$

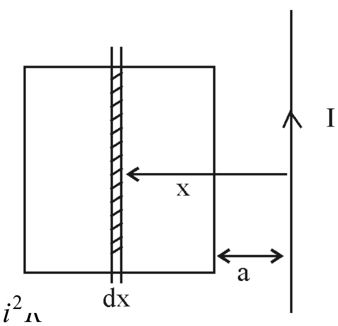


$$E = B\omega\frac{a^2}{2}$$

$$I = \frac{B\omega a^2 4}{2\times 5r}$$

16.(AC) $i = \frac{E}{R}\left[1-e^{-\frac{t}{L}R}\right]$ **17.(AD)** $i = \frac{E}{R}\left[1-e^{-\frac{t}{L}R}\right]$

18.(AC) $B(x) = \frac{\mu_0 I}{2\pi x}$



$d\phi = Badx$, $\phi = \int_a^{2a}\frac{\mu_0 I}{2\pi x}adx = \frac{\mu_0 Ia}{2\pi}\ln 2 = MI$

$M = \frac{\mu_0 a\ln 2}{2\pi}$ Apply len'z law for direction of induced current.

19.(ABD) $i(t) = 4\left[1-e^{-t/1}\right]$, $P_B = Vi = 32\left[1-e^{-t}\right]$ $\Rightarrow$ $P_R = i^2 R$

20.(BCD) $\Delta\phi = BA = 8wb$, $|\varepsilon_{av}| = \frac{8}{0.1} = 80V$

21.(BCD) For $r \le R$, $E2\pi r = \pi r^2\frac{dB}{dt}$

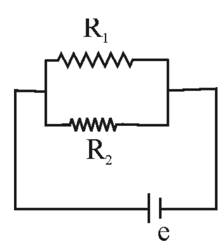
For r > R, $E2\pi r = \pi R^2.\frac{dB}{dt}$

Also apply lenz' law for direction of $\vec{E}$

22.(ABC) $i = \frac{dq}{dt} = 4t$ $V_a - 1\frac{di}{dt} - \frac{q}{2} - 4i = V_d$

23.(AC) $V_a - V_b = V_c - V_b = \frac{1}{2}\omega Bl^2$

24.(A) The equivalent diagram is:
The induced emf across the centre and any on the circumference is:



$$|\vec{e}| = \frac{1}{2}B\omega l^2 = \frac{B\omega r^2}{2}$$

25.(ABC)(A) $t = 0$

$i_L = 0$

$V = \frac{V}{2}$

(B) $t = \infty$

$i = \frac{2V}{3R} \quad i_2 = \frac{V}{3R} \quad \Rightarrow \quad V_A = \frac{V}{3}$

(C) After opening

$i_A = \frac{v}{3R}$

$V_L = V_A + V_R = 2R \times \frac{V}{3R} = \frac{2V}{3}$

$V_A = \frac{V}{3}$

26.(CD) $\varepsilon - iR + \frac{Ldi}{df} + \frac{Mdi}{dt} - \frac{Ldi}{dt} - \frac{Mdi}{dt} = 0$

$\varepsilon - iR = 0$

$i = \frac{\varepsilon}{R}$

Time constant $= \frac{L_{eff}}{R} = 0$ & Power delivered by battery is constant $P = \frac{\varepsilon^2}{R}$

27.(ABCD) $r < R$

$E.2\pi r = \frac{d}{dt}\left[\mu_0 nCt \times \pi r^2\right]$

$r > R$

$E.2\pi r = \frac{d}{dt}\left[\mu_0 nCt \times \pi R^2\right]$

The line charge will produce radially electric field which is perpendicular to induced electric field

28.(ABCD)

At $t = 0$ capacitor behave like conductor & inductor behave like insulator. At steady state capacitor behave like insulator & inductor behave like conductor

29.(ABC) For given situation $\frac{q}{C} + L\frac{di}{dt} = 0$

$\Rightarrow \quad \frac{d^2q}{dt^2} + \frac{q}{LC} = 0 \quad \Rightarrow \quad \frac{d^2q}{dt^2} + \omega^2 q = 0$ compare this equation with $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$

$q \equiv x, i \equiv v, L \equiv m, C \equiv \frac{1}{k}$

Solving equation

$\Rightarrow \quad q = q_0 \cos\omega t$ & $i = -q_0\omega \sin\omega t$

According to given conditions

$\frac{q^2}{2C} = \frac{1}{2}Li^2 \Rightarrow \frac{q_0 \cos^2 \omega t}{2C} = \frac{1}{2}Lq_0^2\omega^2 \sin^2 \omega t \qquad \Rightarrow \cot^2 \omega t = 1$

$\Rightarrow \omega t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} .... \qquad \Rightarrow t = \frac{\pi}{4\sqrt{LC}}, \frac{3\pi}{4\sqrt{LC}}, \frac{5\pi}{4\sqrt{LC}}, \frac{7\pi}{4\sqrt{LC}} ......$

30.(B) $E2\pi \frac{R}{2} = \frac{\pi R^2}{4}\frac{dB}{dt} \quad \Rightarrow \quad F = qE$

31.(C) $\frac{dB}{dt} = 6t^2 + 24 \quad \Rightarrow \quad E2\pi r = \pi r^2 . \frac{dB}{dt} = \frac{r}{2}\frac{dB}{dt}$

32.(B) $\frac{dB}{dt} = 6t^2 + 24$; $\left.\frac{dB}{dt}\right|_{t=1s} = 30\ T/s$ Apply Lenz's law for direction of $\vec{E}$

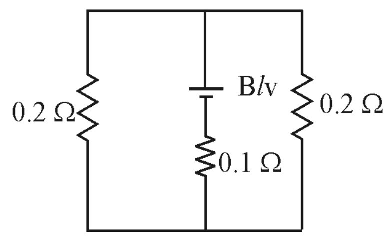
33. **A - s; B - p; C - s; D - s**

34. **A - s; B - q; C - p; D - p**

35.(6) Use maxima-minima

36.(0) $I = I_L + I_C ; I_L = \frac{V}{R}\left[1 - e^{-\frac{tR}{L}}\right]; I_C = \frac{V}{R}e^{-\frac{t}{RC}}$

37.(5) $\varepsilon_{ind} = B\ell V = 1$



38.(8) $\frac{di}{dt} = 10e^{-4t}(-4) = -40e^{-4t}$

$V_L = -L\frac{di}{dt}$

39.(3) $F = ilB$, $i = \frac{BlV}{R}$

40.(8) $B = 0 \quad \Rightarrow \quad t = C/K \quad \Rightarrow \phi = B.\ area = (C - kt)\ \pi a^2$

$E = \frac{d\phi}{dt} \Rightarrow i = \frac{E}{R} = \frac{1}{R}\frac{d\phi}{dt} \Rightarrow q = \left|\int idt\right| = \left|\int \frac{1}{R}\frac{d\phi}{dt}.\ dt\right| = \left|\int \frac{d\phi}{R}\right| = \frac{0 - C\pi a^2}{R} = \frac{C\pi a^2}{R} = \frac{2\pi(2)^2}{\pi} = 8$ coulombs

41.(4) $e = Bv\ell$

$I = \frac{Bv\ell}{R}$

$F = BI\ell = \frac{B^2\ell^2 v}{R}$

$-v\frac{dv}{ds} = \frac{B^2\ell^2 v}{mR}$

$-\int_{v_0}^{0} dv = \frac{B^2\ell^2}{mR}\int_{0}^{s} ds$

$s = 4\,\text{m}$

42.(2) $e = B\ell v = \frac{8}{10}\times 3\times 5 = 12$ Volt $\therefore \quad q = CE(1 - e^{-t/\tau})$

$\therefore \quad 24 = 6\times 12(1 - e^{-t/\tau}) \Rightarrow e^{-t/\tau} = \frac{2}{3}$

$\therefore \quad i = \frac{E}{R}e^{-t/\tau} = \frac{12}{4}\times\frac{2}{3} = 2$

43.(5) Total magnetic flux through a super conductor is equal to zero.

$\Rightarrow \quad \phi_{self} + \phi_{ext} = 0 \quad \Rightarrow \quad |\phi_{self}| = \phi_{ext}$

$$Li = B_0 \pi r^2$$

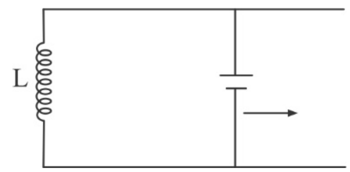
$$i = \frac{B_0 \pi r^2}{L}$$

44.(4) $L\frac{di}{dt} = vB\ell$

Amperes force law opposes velocity

$$iB\ell = -m\frac{dv}{dt}$$

Figure



$$\frac{di}{dt} = -\frac{m}{B\ell}\frac{d^2v}{dt^2}$$

So $\frac{d^2v}{dt^2} + \frac{B^2\ell^2}{mL}v = 0$ *i.e.*, $\omega = \frac{B\ell}{\sqrt{mL}}$

45.(2) When velocity is maximum then net force acting on the conductor is zero.

AC CIRCUIT AND EM WAVES

1.(C) Resultant voltage = 200 volt

Since V_1 and V_3 are out of phase 180°, the resultant voltage is equal to V_2 $\therefore$ V_2 = 200 volt

2.(C) $i_{1rms} = \dfrac{E_{rms}}{\sqrt{X_c^2 + R_1^2}} = \dfrac{130}{13} = 10A$; $i_{2rms} = \dfrac{E_{rms}}{\sqrt{X_L^2 + R_2^2}} = 13A$

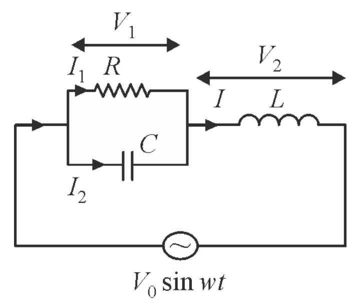
Power dissipated = $i_{1rms}^2 R_1 + i^2{}_{2rms} R_2 = 10^2 \times 5 + 13^2 \times 6$ W = power delivered by battery = 500 + 169×6 W

3.(A) $I = 10\ A$, $V = 1000$ volt, $V_R = IR = 1000\ V$ it, $V_c = 200\ V$

$V^2 = (V_L - V_C)^2 + V_R^2$

$\Rightarrow \quad 1000^2 = (V_L - 200)^2 + 1000^2 \quad \Rightarrow \quad V_2 = 200V$ (Resonance condition)

4.(B) $\tan\theta = \dfrac{I_2}{I_1} = \dfrac{R}{X_C}$



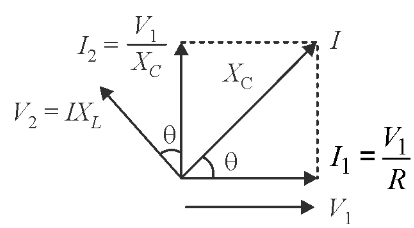
$I^2 = I_1^2 + I_2^2 = V_1^2\left(\dfrac{1}{R^2} + \dfrac{1}{X_C^2}\right)$

$V_0^2 = V_1^2 + V_2^2 + 2V_1V_2\cos(90+\theta)$

$= \dfrac{I^2}{\frac{1}{R^2} + \frac{1}{X_C^2}} + I^2X_L^2 - 2\cdot\dfrac{I}{\sqrt{\frac{1}{R^2}+\frac{1}{X_C^2}}} IX_L \dfrac{R}{\sqrt{R^2 + X_C^2}}$

$V_0^2 = I^2\left(\dfrac{R^2X_C^2}{R^2+X_C^2} + X_L^2 - \dfrac{2X_L\ RX_C\ R}{R^2+X_C^2}\right)$

$= I^2\left[\dfrac{R^2}{R^2+X_C^2}\left(X_C^2 - 2X_LX_C\right) + X_C^2\right]$



$X_C^2 - 2X_LX_C = 0 \qquad \Rightarrow X_C = 2X_L$

$\dfrac{1}{\omega C} = 2\omega L \qquad \Rightarrow \qquad \omega^2 = \dfrac{1}{2LC}$

5.(D) $E_0 = CB_0$

6.(C)

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\varepsilon_0 \dfrac{d\Phi_E}{dt}$ **. . . (i)**

In this case, no current is flowing and hence I = 0

Due to symmetry, $B = \vec{B}$ is constant

Hence

$\oint \vec{B}.d\vec{l} = 2\pi rB$

Substituting in equation (i)

$2\pi rB = \mu_0\varepsilon_0 \dfrac{d\Phi_E}{dt}$ **. . . (ii)**

$\phi_E = \pi R^2 E$

and

$\mu_0\varepsilon_0 = \dfrac{1}{c^2}$

Hence, from equation (ii)

$B = \dfrac{\mu_0\varepsilon_0 \times \pi R^2}{2\pi r}\dfrac{dE}{dt} = \dfrac{1}{c^2} \times \dfrac{0.7^2}{2\times 1.2} \times 5\times 10^{12} = 1.1\times 10^{-5}\,T$

7.(ACD) $\sqrt{V_R^2 + \left(V_C - V_L\right)^2} = 130 \Rightarrow V_R = 50V$, $\sqrt{V_R^2 + V_L^2} = 100$

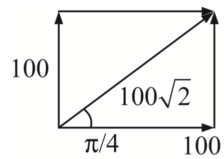
8.(ABC) $X_C = \dfrac{V_C}{I}; V_L = IX_L$

9.(AC) $I_{upper} = \dfrac{20}{100\sqrt{2}};\ +\dfrac{\pi}{4}$ ahead of voltage

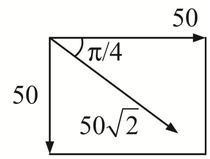
$I_{lower} = \dfrac{20}{50\sqrt{2}};\ -\dfrac{\pi}{4}$ behind voltage

$I = \sqrt{I_1^2 + I_2^2} = \sqrt{\dfrac{1}{10}} = 0.3A$

$V_{100\Omega} = \dfrac{20}{100\sqrt{2}} \times 100 = 10\sqrt{2}$



Capacitive circuit



Inductive circuit

10.(ABCD) $x_L > x_C$

11.(AC) $X_C = \dfrac{2}{5}X_L$

$Z = \sqrt{R^2 + (X_L - X_C)^2}$, $\dfrac{R}{Z} = 0.8$

Now solving get answer.

12.(AC) $\omega_r = \dfrac{1}{\sqrt{LC}} = 50\pi,\qquad f_r = 25\,Hz$

Band width $= \dfrac{R}{L} = \dfrac{R}{\frac{1}{25\pi}} = 250\pi$; $Q = \dfrac{\omega_r}{\text{band width}} = \dfrac{50\pi}{250\pi} = \dfrac{1}{5}$; $\omega_2 - \omega_1 = 250\pi$ and $\omega_1\omega_2 = \omega_r^2$

$(\omega_2 + \omega_1) = \sqrt{(\omega_2 - \omega_1)^2 + 4\omega_r^2}$; $= \sqrt{(250\pi)^2 + 4(2500\pi^2)}$; $= \sqrt{72500\pi^2} = 10\pi\sqrt{725}$

13.(D) 14.(C) 15.(A)

When connected with the DC source $R = \dfrac{12}{4} = 3\Omega$

When connected to ac source $I = \dfrac{V}{Z}$ $\qquad \therefore 2.4 = \dfrac{12}{\sqrt{3^2 + \omega^2 L^2}} \Rightarrow L = 0.08\,H$

Using $I_{rms}V_{rms}\cos\phi = \frac{V_{rms}^2}{Z}\cos\phi = \frac{V_{rms}^2 R}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = 24W$

16.(B) **17.(C)** **18.(D)**

19. **(A – r) ; (B – p) ; (C – q) ; (D – s)**

20.(2) $\tan 60° = \frac{X_L}{R} \Rightarrow X_L = 100\sqrt{3}\,\Omega = X_C$ $\quad \Rightarrow Z = R$

21.(5) $I = \frac{V_R}{R} = \frac{20}{4} = 5A$

22.(5) $\cos\phi = \frac{1}{\sqrt{2}}$, $\phi = 45°$, $\tan\phi = 1 \Rightarrow L\omega = R$; $\omega = \frac{R}{L} = \frac{3}{6\times 10^{-3}}$ rad/sec = 500 rad/sec.

23.(4) AC ammeter shows rms current

So, when both currents are flown simultaneously, AC ammeter gives

$I_{AC} = \sqrt{6^2 + 8^2} = 10$ & DC ammeter gives,

$I_{DC} = 6$

Difference in readings $= 10 - 6 = 4$

24.(4) $P = V_1 i_1 = V_2 i_2$

$10\times 10^3 = 25\times V_2$

$V_2 = \frac{10^4}{25}$

Now $V_1 = \frac{n_1}{n_2}\times V_2 = \frac{8}{1}\times\frac{10^4}{25}V$

25.(5) $I_{rms} = \frac{E_{rms}}{Z} = \frac{E_v}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = 0.2$ amp

$\therefore \quad H = I^2 Rt = 2\times 10$ or $(0.2)^2\times 100\times t = 20$ $\quad\therefore\quad t = \frac{20}{(0.2)^2\times 100} = 5\sec$

26.(2) $z_{RL} = \sqrt{R^2 + \omega^2 L^2}$

$z_{RLC} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$\cos\varnothing_{RL} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$

According to question

$(\omega L)^2 = \left(\omega L - \frac{1}{C\omega}\right)^2 \quad\Rightarrow\quad C = \frac{1}{2\omega^2 L}$

27.(2) Displacement current $i_d = \varepsilon_0 \dfrac{d\phi_e}{dt} = \varepsilon_0 \dfrac{d(\vec{E}.\vec{A})}{dt} \Rightarrow i_d \propto A$

$\because \quad i_d' = \dfrac{A'}{A} \times i_d = \dfrac{0.3}{0.6} \times 1 = \dfrac{1}{2} A$

28.(14) We know that $i_d = \varepsilon_0 \dfrac{d\phi_E}{dt} = \varepsilon_0 A.\dfrac{dE}{dt}$

$\Rightarrow \quad \varepsilon_0 A.\dfrac{dE}{dt} = i_d$

$\Rightarrow \quad \dfrac{dE}{dt} = \dfrac{i_d}{\varepsilon_0 A}$

$\Rightarrow \quad \dfrac{dE}{dt} = \dfrac{17.7}{8.85 \times 10^{-12} \times 100 \times 10^4}$

$\Rightarrow \quad \dfrac{dE}{dt} = 2 \times 10^{14}\ N\ C^{-1}\ s^{-1}$

By comparing

$2 \times 10^{14}\ N C^{-1} = 2 \times 10^{N}\ N C^{-1}\ s^{-1}$

So the value of N is 14

29.(2) $I = \dfrac{1}{2\mu_0} B_0^2 c$

Given that the power of the point source is P. Intensity of radiation due to the point source at a distance *r* will be,

$I = \dfrac{P}{4\pi r^2}$

$\Rightarrow \quad \dfrac{P}{4\pi r^2} = \dfrac{1}{2\mu_0} B_0^2 c$

$\Rightarrow \quad \dfrac{2\mu_0 P}{4\pi r^2 c} = B_0^2$

$\Rightarrow \quad B_0 = \sqrt{\dfrac{\mu_0 P}{2\pi r^2 c}}$

By comparing our obtained value, we can say that the value of *k* will be 2.

30.(115) Capacitive reactance

$X_C = \dfrac{1}{\omega C}$

$= \dfrac{1}{300 \times 100 \times 10^{-12}}$

$= \dfrac{10^8}{3} \Omega$

There is only capacitance in the circuit

$\therefore \quad i_{rms} = \dfrac{V_{rms}}{X_C}$

$= \dfrac{230}{(10^8 / 3}$)

$= 6.9 \times 10^{-6} A$

Hence i_d is displacement current and I the conduction current. Magnetic field at a distance r from the axis,

$$B = \frac{\mu_0}{2\pi} \frac{i_d}{R^2} r$$

$\therefore \quad B_{rms} = \dfrac{\mu_0}{2\pi} \dfrac{i_{rms}}{R^2} r \qquad (i_d = i = i_{rms})$

Substituting the values, we have

$$B_{rms} = \frac{(2\times10^{-7})(6.9\times10^{-6})}{(6\times10^{-2})^2}(3\times10^{-2})$$

$= 1.15 \times 10^{-11} T$

RAY OPTICS AND WAVE OPTICS

1.(B) $\sin\theta_c = \frac{\sqrt{3}}{2}$

$\theta_c = 60°$ $\quad\therefore\quad$ $\cos 60° = \frac{R}{R+d}$

$d = R.$

2.(A) Coordinate of image formed by lens f

$(f, 0)$

For lens 2 :

$u = -2f,\ h_0 = -d$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$v = 2f$

$m = \frac{v}{u} = \frac{2f}{-2f} = -1$

$h_i = +d$

Coordinate of image $(3f + 2f,\ d + d)$

3.(C) For $y = 0$ For $x = 0$

$x = 3$ $\quad y = 1$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{x} - \frac{1}{3} = \frac{1}{15}$

$x = \frac{5}{2}$

Equation of line passing through (0, 1) & $\left(\frac{5}{2}, 0\right)$

$y = -0.4x + 1$

4.(D) The effective distance of the screen = 2D + 2D = 4D $\quad\therefore\quad$ Fringe-width $= \frac{4D}{d}\lambda$

5.(C) $\hat{I}\cdot\hat{N} = \hat{r}\cdot\hat{n}$

6.(B) Final Intensity obtained

$I' = I_0 - 0.578125I = 0.421875I_0$

Using Malus Law, $I' = I_0(\cos^2\theta)^n$ $\quad\Rightarrow\quad$ $0.421875I_0 = I_0(\cos^2\theta)^n$

$\Rightarrow\quad (0.75)^3 = (\cos^2\theta)^n$ $\quad\Rightarrow\quad$ $n = 3$ and $\cos = \frac{\sqrt{3}}{2}$ $\Rightarrow\ \theta = 30°$

Angle rotated $\phi° = n\theta = 3\times 30° = 90°$

7.(B) There is no change of phase in the transmitted ray due to difference in refractive index. In the reflected ray, phase change occurs when a ray is reflected from an optically denser medium.

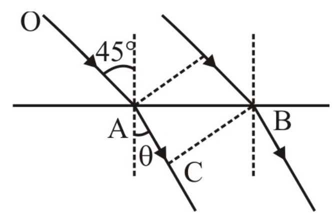
8.(D) Width of central diffraction maximum $=\dfrac{2\lambda D}{b}$

$\Rightarrow \quad 5.2\times10^{-3} = \dfrac{2\times500\times10^{-9}\times2.08}{b} \quad \Rightarrow \quad b=0.4\ mm$

and $N=\dfrac{\dfrac{2\lambda D}{b}}{\dfrac{\lambda D}{d}} \Rightarrow d=nb$

So, $d=11\times0.4\,mm = 4.4\ mm$

9.(D) $AB=\sqrt{2}$ cm



From refraction at air-water boundary,

$(1)\left(\dfrac{1}{\sqrt{2}}\right)=\left(\sqrt{2}\right)\sin\theta$ or $\theta=30°$ $\quad\therefore\quad BC=\dfrac{\sqrt{3}}{\sqrt{2}}$ cm.

10.(C) $3\lambda\cos\theta=2\lambda$ (θ is the angular position of the point)

$\therefore \quad \cos\theta=\dfrac{2}{3} \qquad \therefore \quad y=\dfrac{\sqrt{5}}{2}D$

11.(B) $\sin\theta_c=\dfrac{\mu_2}{\mu_1}$

$1-\cos^2\theta_C=\left(\dfrac{\mu_2}{\mu_1}\right)^2$

$1-\left(\hat{n}\cdot\hat{p}\right)^2=\left(\dfrac{\mu_2}{\mu_1}\right)^2 \quad ; \quad 1-\left[\dfrac{4}{\sqrt{25}}\right]^2=\left(\dfrac{\mu_2}{\mu_1}\right)^2 \Rightarrow \dfrac{\mu_2}{\mu_1}=\dfrac{3}{5} \quad \Rightarrow \quad \mu_2=\dfrac{3\sqrt{3}}{5}$

12.(C) For required condition the light ray should be parallel to principal axis after refraction through curved surface of lens.

$\dfrac{3/2}{\infty}-\dfrac{1}{-x}=\dfrac{3/2-1}{+20} \Rightarrow x=40\ cm$

13.(A) Optical path difference $=\left(S_2O\right)n_2-\left[\left(S_1O-t\right)n_2+tn_3\right]$

$\Rightarrow \quad t\left(n_2-n_3\right)\ or\left(t\left(n_2-n_2\right)\right)$ which ever is positive

Wave length in vacuum $= n_1\lambda_1$

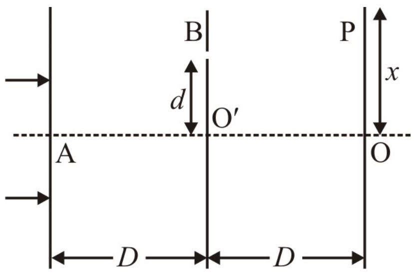
$\Rightarrow \quad \text{phase diff.} =\dfrac{2\pi\left(\text{optical path diff.}\right)}{\text{Wavelength in vaccum}}=\dfrac{2\pi t\left(n_2-n_3\right)}{n_1\lambda_1}$

14.(C) $I=I_{max}\,cos^2\dfrac{\pi x}{\omega}$

$\Rightarrow \quad I_{max} = I_{max} cos^2 \frac{\pi x}{\omega} \Rightarrow cos\frac{\pi x}{\omega} = \pm\frac{1}{2} \quad \Rightarrow \quad \frac{\pi x}{\omega} = \frac{\pi}{3}$ . . . . . . for smallest $x \Rightarrow x = \frac{\omega}{3} = \frac{\lambda D}{3d}$

15.(A) $\Delta r = d\sin\theta + \left(\frac{\mu_s}{\mu_\omega} - 1\right)t = \frac{9}{16}mm \neq n\lambda$ **16.(AC)** **17.(ABCD)**

18.(BD)



There is a dark fringe at O if the path different $\delta = ABO - AO'O = \frac{\lambda}{2}$

$= 2\sqrt{D^2 + d^2} - 2D = \frac{2d^2}{2D} = \frac{d^2}{D} = \frac{\lambda}{2} \quad \Rightarrow d_{\min} = \sqrt{\frac{\lambda D}{2}}$

The bright fringe is formed at P if the path different $\delta' = AO'P - ABP = \lambda$

$= D + \sqrt{D^2 + x^2} - \sqrt{D^2 + d^2} - \sqrt{D^2 + (x-d)^2} = \lambda = \frac{x^2}{2D} - \frac{d^2}{2D} = \frac{(x^2 + d^2 - 2xd)}{2D} = \lambda$

Given, $d = d_{\min}$ Solving, $x = d_{\min} = \sqrt{\frac{\lambda D}{2}}$

19.(ABCD)Δx at O = d [path difference is maximum at O]

So, if $d = \frac{7\lambda}{2}$, O will be minima

$d = \lambda$, O will be maxima

$d = \frac{5\lambda}{2}$, O will be minima and hence intensity is minimum.

If $d = 4.8\lambda$, then total 10 minimas can be observed on screen, 5 above O and 5 below O, which correspond to

$\Delta x = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}, \pm\frac{7\lambda}{2}, \pm\frac{9\lambda}{2}$

20.(AD) If the amplitude due to two individual sources at point P is A_0 and $3A_0$ then the resultant amplitude at P, will be :

$A = \sqrt{A_0^2 + (3A_0)^2 + 2(A_0)(3A_0)\cos\frac{\pi}{3}} = \sqrt{13}A_0$

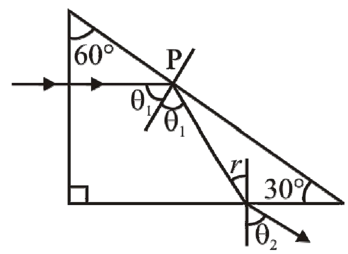
Resultant intensity, $I \propto 13A_0^2$

21.(BC) Shift $d = \left[1 - \frac{\cos i}{\sqrt{n_1^2 - \sin^2 i}}\right]t_1 \sin i + \left[1 - \frac{\cos i}{\sqrt{n_2^2 - \sin^2 i}}\right]t_2 \sin i$

$i = 37° n_1 = \frac{3}{2} t_1 = 4.5 \text{ cm} \, n_2 = \frac{4}{3}, \, t_2 = 2 \text{ cm}$

$$d = \left[1 - \frac{0.8}{\sqrt{\left(\frac{3}{2}\right)^2 - 0.6^2}}\right]\frac{9}{2} \times 0.6 + \left[1 - \frac{0.8}{\sqrt{\left(\frac{4}{3}\right)^2 - 0.6^2}}\right] 2 \times 0.6 = 1.129 + 0.39 = 1.5 \text{ cm} \left[d = d_1 + d_2\right]$$

22.(AC) From the geometry of prism : $\theta_1 = 60$, $r = 30$

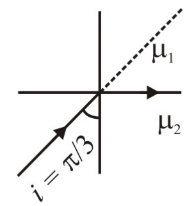


Then apply Snell's law : $\frac{5}{3}\sin r = \frac{4}{3}\sin\theta_2 \; \frac{5}{3} \times \frac{1}{2} = \frac{4}{3}\sin\theta_2$

$\Rightarrow \qquad \sin\theta_2 = \frac{5}{8} \Rightarrow \theta_2 = \sin^{-1}\left(\frac{5}{8}\right)$.

Total internal reflection at the point P is only possible if $\mu_P > \mu_m$

23.(BCD)



Apply Snell's law : $\mu_2 \sin i = \mu_1 \sin r \Rightarrow \sin i = k \sin r$

From the given graph, angle of deviation decreases and becomes zero at $k = k_2$

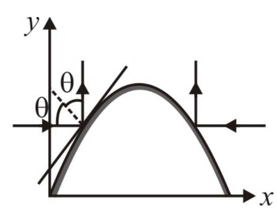
Hence, $\theta_1 = |r - i| = \frac{\pi}{6}$ (By geometry)

$\Rightarrow \qquad$ at $k = k_2$, $\theta = |r - i| = 0$ means, $k_2 = 1$.

$\Rightarrow \qquad$ when $k = \infty$, $r = 0$, by the Snell's law, $\theta_2 = |r - i| = i = \frac{\pi}{3}$

$\Rightarrow \qquad k_1$ = must be less than k_2 from the given graph.

24.(BD) There are two possibilities $2\theta = 90$ (According to the question) $\theta = 45$



$\frac{dy}{dx} =$ Slope of the tangent = tan θ $\quad \frac{dy}{dx} = \tan 45 = 1$

Now, $\frac{dy}{dx} = \frac{2L}{\pi}\left(\frac{\pi}{L}\right)\cos\left(\frac{\pi x}{L}\right)$ (From the given equation)

$\Rightarrow \qquad \frac{dy}{dx} = 2\cos\frac{\pi x}{L} \Rightarrow \cos\frac{\pi x}{L} = \frac{1}{2} \left(\text{Since, } \frac{dy}{dx} = 1\right) \qquad \Rightarrow \qquad \frac{\pi x}{L} = \frac{\pi}{3}, \frac{2\pi}{3} \qquad \Rightarrow x = \frac{L}{3}, \frac{2L}{3}$

25.(BCD) From the displacement method : $h_0 = \sqrt{h_1 h_2} = \sqrt{9 \times 4} = 6$ cm. Hence, option (C) is correct

$$m_1 = \frac{h_1}{h_0} = \frac{9}{6} = \frac{3}{2} \qquad ; \qquad m_2 = \frac{h_2}{h_0} = \frac{4}{6} = \frac{2}{3}$$

From the displacement method :

$$|m_1 - m_2| = \frac{d}{f} \qquad \frac{3}{2} - \frac{2}{3} = \frac{d}{f} \qquad \ldots \text{(i)}$$

$$f = \frac{D^2 - d^2}{4D} = \frac{90^2 - d^2}{4 \times 90} \qquad \ldots \text{(ii)}$$

On solving equation (i) and (ii) we get, $d = 18$ cm, $f = 21.6$ cm

Hence, option (D) is correct.

For the position of object :

$$x_2 - x_1 = d = 8,\ x_2 + x_1 = D = 90$$

$\therefore$ $x_1 = 36$ cm, $x_2 = 54$cm . Here x_1 and x_2 be the position of object for two positions of the lens.

Hence, option (B) is correct.

26.(ABCD) $D = 96\ cm$

$\frac{m_1}{m_2} = 4$, also $m_1 m_2 = 1$ $\Rightarrow$ $m_1 = 2,\ m_2 = \frac{1}{2}$

Also $\frac{V_1}{u_1} = m_1 \Rightarrow V_1 = 2u_1$

Also $V_1 + u_1 = 96$

$\Rightarrow$ $3u_1 = 96$ $\Rightarrow$ $u_1 = 32$ $\Rightarrow$ $V_1 = 64$ $\Rightarrow$ $u_2 = 64,\ V_2 = 32$

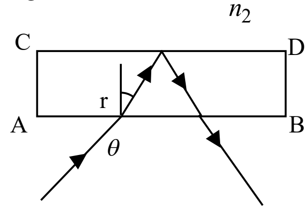
Distance between two positions $= u_2 - u_1 = 32 = L$

Focal length $= \frac{D^2 - L^2}{4D} = \frac{96^2 - 32^2}{4 \times 96} = 64/3\ cm$

For shorter image, $V = V_2 = 32\ cm$

27.(AC) $\theta > sin^{-1}\frac{n_1}{n_2}$ then θ will also greater then $sin^{-1}\frac{n_3}{n_2}$ if $n_3 < n_1$

If $n_3 > n_1$ then



At AB, $n_3 \sin r = n_2 \sin\theta$

$\Rightarrow \quad \sin r = \frac{n_2}{n_3} . \sin\theta$

$\geq \frac{n_2}{n_3} . \sin\left\{\sin^{-1}\frac{n_1}{n_2}\right\} = \frac{n_1}{n_3}$

$\Rightarrow \quad r \geq \sin^{-1}\frac{n_1}{n_3} \Rightarrow$ *TIR* at *CD*

28.(AC) $\frac{1}{f_{air}} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$; $\frac{1}{f_{water}} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

From these two equations we get,

$$f_{water} = 4f_{air} = 4f$$

In air object was inverted, real and magnified. Therefore, object was lying between f and $2f$. Now the focal length has changed two $4f$. Therefore, the object now lies between pole and focus. Hence, the new image will be virtual and magnified.

29.(AC) The intensity of light is $I(\theta) = I_0 \cos^2\left(\frac{\delta}{2}\right)$

Where $\delta = \frac{2\pi}{\lambda}(\Delta x) = \left(\frac{2\pi}{\lambda}\right)(d \sin\theta)$

(i) For $\theta = 30°$

$\lambda = \frac{c}{v} = \frac{3\times10^8}{10^6} = 300\,m$ and $d = 150\ m$

$\delta = \left(\frac{2\pi}{300}\right)(150)\left(\frac{1}{2}\right) = \frac{\pi}{2}$ $\quad\therefore\quad \frac{\delta}{2} = \frac{\pi}{4}$

$\therefore I(\theta) = I_0 \cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2}$ (Option A)

(ii) For $\theta = 90°$

$\delta = \left(\frac{2\pi}{300}\right)(150)(1) = \pi$ Or $\frac{\delta}{2} = \frac{\pi}{2}$ and $I(\theta) = 0$

(iii) For $\theta = 0°, \delta = 0$ *or* $\frac{\delta}{2} = 0$

$I(\theta) = I_0$ (Option C)

30.(AD) Final image is formed at infinity if the combined focal length of the two lenses (in contact) becomes 30 *cm* or

$$\frac{1}{30} = \frac{1}{20} + \frac{1}{f}$$

$$f = -60\ cm$$

i.e., when another concave lens of focal length 60 *cm* is kept in contact with the first lens.

Similarly, let μ be the refractive index of a liquid in which focal length of the given lens becomes 30 *cm*. Then

$$\frac{1}{20} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots\ldots(1)$$

$$\frac{1}{30} = \left(\frac{3/2}{\mu} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots\ldots(2)$$

From equations (1) and (2), we get

$$\mu = \frac{9}{8}$$

31.(BD) $2\mu_1 t = n\lambda$

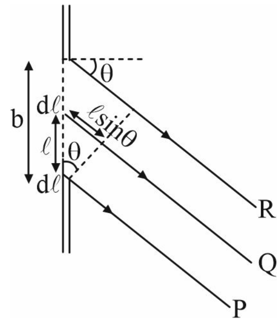
$t = \frac{n\lambda}{2\mu_1} = n\frac{640\times3}{2\times4} = 240\,n$

$t = 240\,nm,\ 480\,nm, \ldots.$

32.(BD) The condition for getting maxima is $d\sin\theta = m\lambda$. The wavelength of electron will be given as $\lambda = h/mv$.

The distance between successive maxima will increase if θ becomes smaller. For this either d should be increased or λ should be decreased. For the latter we must increase the voltage V.

33.(B) The disturbances from arbitrary ray Q reaching on screen will be



$$y_Q = \frac{A\,d\ell}{b}\sin\left(\omega t + \frac{2\pi\ell\sin\theta}{\lambda}\right)$$

Using principle of superposition we can say the resultant disturbance at any point at angle θ will be

$$y_{net} = \int_0^b \frac{A}{b}\,d\ell\sin\left(\omega t + \frac{2\pi\ell\sin\theta}{\lambda}\right) = \frac{A\lambda/b}{\sin\theta}\,d\ell\sin\left(\omega t + \frac{2\pi\ell\sin\theta}{\lambda}\right)$$

$$= \frac{A\lambda/b}{2\pi\sin\theta}\left\{\cos\omega t - \cos\left(wt + \frac{2\pi b\sin\theta}{\lambda}\right)\right\}$$

$$y_{net} = \frac{A\lambda}{\pi b\sin\theta}\sin\left(\frac{\pi b\sin\theta}{\lambda}\right)\sin\left(\omega t + \frac{\pi b\sin\theta}{\lambda}\right)$$

Resultant Amplitude $(A_{net}) = \dfrac{A\lambda}{\pi b\sin\theta}\sin\left(\dfrac{\pi b\sin\theta}{\lambda}\right)$

And $I_{net} \propto (A_{net})^2 \quad \Rightarrow \quad I_{net} \propto A^2\left(\dfrac{\lambda}{\pi b\sin\theta}\right)^2\sin^2\left(\dfrac{\pi b\sin\theta}{\lambda}\right) \quad \Rightarrow \quad I_{net} = \dfrac{I_0\sin^2\left(\dfrac{\pi b\sin\theta}{\lambda}\right)}{\left(\dfrac{\pi b\sin\theta}{\lambda}\right)^2}$

34.(D) For maxima $\dfrac{\pi b\sin\theta}{\lambda} = \pm 1$

$$\Rightarrow \quad \frac{\pi b\sin\theta}{\lambda} = \pm\left(m + \frac{1}{2}\right)\pi \qquad \text{(m = whole number)}$$

$$\Rightarrow \quad b\sin\theta = \pm\left(m + \frac{1}{2}\right)\lambda$$

For 2^{nd} maxima, m = 2

$$\Rightarrow \quad b\sin\theta = \pm\,\frac{5}{2}\lambda$$

$$(I_{nct})_2 = \frac{I_0\sin^2\left(\frac{5\pi}{2}\right)}{\left(\frac{5\pi}{2}\right)^2} = 0.016 I_0$$

35.(B) For 2^{nd} maximum

$$b\sin\theta = \pm\frac{5}{2}\lambda \;\Rightarrow\; b\tan\theta \approx \pm\frac{5\lambda}{2}$$

$$\Rightarrow \quad \frac{by}{D} = \pm\frac{5\lambda}{2}$$

$$\Rightarrow \quad y = \pm\,\frac{5\lambda D}{2b}$$

36.(B) Focal length of plano-convex lens

$$f = 20\text{cm}$$

$$\frac{1}{f_{eq}} = -\frac{2}{f}$$

$$f_{eq} = -\frac{20}{2} = -10 \text{ cm}$$

37.(B) As mass of lens = mass of particle

Time period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{kx_0}{gk}} = 2\pi\sqrt{\frac{x_0}{g}} = .2$

At time $t = \frac{T}{2}$, lens will come to same position (mean).

Distance travelled by particle in time $\frac{T}{2}$

$$u = \frac{1}{2}gt^2 = \frac{1}{2}\times 10\times(0.1)^2 = 0.05\text{m} \quad \Rightarrow \quad u = 5 \text{ cm}$$

Velocity of particle $v_p = g\cdot t = 10\times 1 = 1$ m/s

Velocity of lens $v = 10$ m/s

Location of image $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

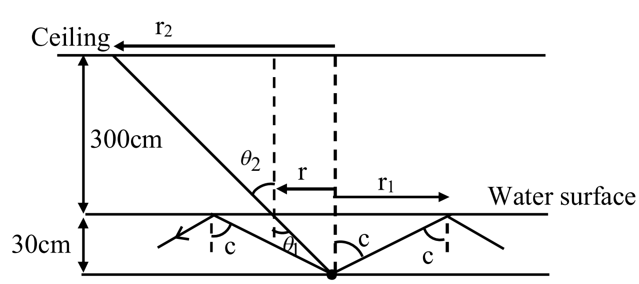
$$\frac{1}{v}+\frac{1}{(-s)}=\frac{1}{-10} \quad \Rightarrow \quad v = +10 \text{ cm}$$

Velocity of image : $v_{i/l} = \left(\frac{v}{u}\right)^2 v_{o/l}$

$$v_{i/l} = \left(\frac{10}{5}\right)^2(10-1) \quad \Rightarrow \quad v_{i/l} = 36 \text{ m/s}$$

38.(C)

39-41. **39.(B)** **40.(C)** **41.(B)**



$$\sin c = \frac{1}{\mu} = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \cos c = \frac{\sqrt{2}}{\sqrt{3}}$$

$$r_1 = 30\tan c = \frac{30}{\sqrt{2}}\,cm = 15\sqrt{2}\,cm$$

$r = 10\sqrt{3} < r_1$ (Hence shadow of ring will be formed on roof)

Radius of shadow = $r_2 = r + 300\tan\theta_2 = 10\sqrt{3} + 300\tan\theta_2$

Now from shell's Law $\sqrt{3}\sin\theta_1 = \sin\theta_2$ $\left(\tan\theta_1 = \frac{10\sqrt{3}}{30} = \frac{1}{\sqrt{3}}\right)$ $\Rightarrow$ $\sqrt{3}\times\frac{1}{2} = \sin\theta_2$ $\Rightarrow$ $\tan\theta_2 = \sqrt{3}$

Hence radius of shadow = $10\sqrt{3} + 300\sqrt{3} = 310\sqrt{3}\,cm$

$r_{max} = r_1$ for which light will come out of liquid surface and shadow of ring will be formed on ceiling

42. **[A-p, q] [B-q, r, s] [C-p, q, r] [D-q, r, s]**

43. **[A-p, t] [B-r, t] [C-s, t] [D-s, t]**

44. **[A-q, t] [B-p, r, t] [C-p, q, s, t] [D-p, r]**

45. **[A-p, r] [B-q] [C-p, s] [D-p, r]**

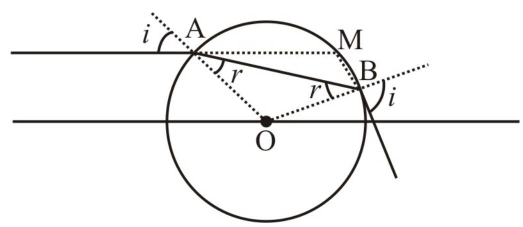
At centre intensity will be maximum for both wavelengths.

For maxima: $\frac{d\ell}{D} = n\lambda \ (n = 0, 1, 2.......)$

$\Rightarrow \quad \ell = \frac{n\lambda D}{d} = \left(0.2\,mm,\ 0.4\,mm, ...\text{for } 4000\,A^\circ\right) = \left(0.4\,mm,\ 0.8\,mm, ...\text{for } 8000\,A^\circ\right)$

For minima: $\ell = \left(0.1\,mm,\ 0.3\,mm, ...\text{for } 4000\,A^\circ\right) = \left(0.2\,mm,\ 0.6\,mm, ...\text{for } 8000\,A^\circ\right)$

46.(2) Ray diagram is shown



$\angle AOB = \pi - 2r$

Now, $\angle AMB = \pi - \frac{1}{2}\angle AOB$ (why?)

$\therefore \quad \angle AMB = \pi - \frac{1}{2}(\pi - 2r) = \frac{\pi}{2} + r$

$\therefore$ In ΔAMB ; $i - r + \frac{\pi}{2} + r + i - r = \pi$

$\Rightarrow$ Now $\frac{\sin i}{\sin r} = \mu \quad \Rightarrow \quad \sin i = \sqrt{3}\sin\left(2i - \frac{\pi}{2}\right)$

$\Rightarrow \quad \sin i = \frac{\sin i}{\sin r}\cos 2i \quad \Rightarrow \quad 2\sqrt{3}\sin^2 i - \sin i - \sqrt{3} = 0 \quad \Rightarrow \quad \sin i = \frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{3}}$.

Rejecting the negative sign

We get $i = 60°$.

47.(4) $\frac{1}{f_{\text{air}}} = -\frac{1}{50} + \frac{\mu_w}{u}$ … (i)

and $\frac{1}{f_w} = -\frac{1}{40 \times \mu_w} + \frac{1}{u}$ … (ii) $\quad \therefore \quad f_w = 4 f_{\text{air}}$

48.(4) For plane surface up $\mu = \frac{4}{3}$

For curved surface up

$\frac{1}{v} - \frac{\mu}{u} = \frac{(1-\mu)}{R}$

$v = -\frac{25}{8}, u = -4$

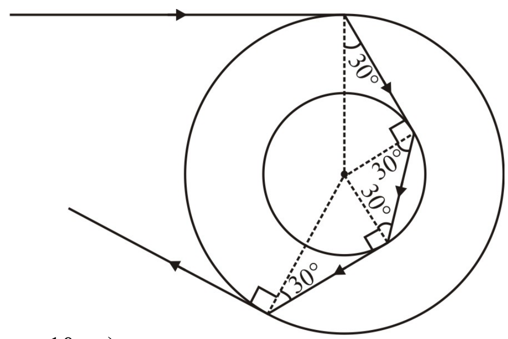
$R = 25$ cm = $\frac{1}{4}$ m.

49.(8) $\sin\theta_c = \frac{2\mu}{3}$

At face AC, i is 60°

$i > \theta_c$

50.(4)



51.(16) In one case image is virtual (u = -10cm)

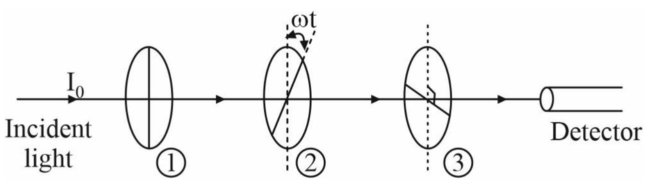
In another case image is real (u = -40cm)

$v_1 = \frac{uf}{u+f} = \frac{-10f}{-10+f}$; $v_2 = \frac{-40f}{-40+f}$

In both situations, sign convention is opposite.

$\Rightarrow \quad v_2 = v_1 \quad \Rightarrow \quad v_2 = \frac{-10f}{-10+f} = \frac{40f}{-40+f}$; $f = 16cm$

52.(4)



After crossing polaroid 1, intensity of beam is $I_1 = \frac{I_0}{2}$

After crossing polaroid 2, intensity of beam is $I_2 = I_1 \cos^2 \omega t$

$\Rightarrow \quad I_2 = \frac{I_0}{2}\cos^2 \omega t$

After passing through 3rd polaroid, intensity of beam will be

$I_3 = I_2 \cos^2\left(\frac{\pi}{2} - \omega t\right) = I_2 \sin^2 \omega t \quad \Rightarrow \quad I_3 = \frac{I_0}{2}\cos^2 \omega t \sin^2 \omega t$

$I_3 = \frac{I_0}{8}(\sin 2\omega t)^2 = \frac{I_0}{16}(1 - \cos 4\omega t)$

So, $a = 16$ & $b = 4 \Rightarrow \frac{a}{b} = 4$

53.(30) $m_1 m_2 = \frac{f}{f-x} \times \frac{f'}{\frac{xf}{x-f} - d + f'} = \frac{f\ f'}{-xf - d(f-x) + f'(f-x)}$

$-xf + dx - xf' = 0$; $f + f' = d = 30cm$

54.(8) $\frac{1}{f}=(\mu_{rel}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ and $\frac{1}{f_{meq}}=\frac{1}{f_{m'}}-2\left(\frac{1}{f}\right)$

55.(1) From the information given, $f_{1st\ lens}=30cm$

From lens formula (for second lens)

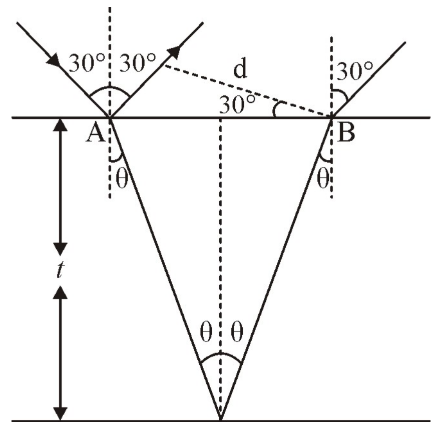
$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{22-x}-\frac{1}{30-x}=\frac{1}{30}$

56.(3) Given $d=4\sqrt{3}cm\ \sin 30°=\frac{17}{16}\sin\theta \sin\theta=\frac{8}{17}$

$AB=d\sec 30°=8\,cm$

$\tan\theta=\frac{4}{t} \Rightarrow \frac{8}{15}=\frac{4}{t}$

$\Rightarrow \quad t=7.5cm=75\,mm$



57.(8) $1.5\times10^{-2}=\frac{n\lambda D}{d}=\frac{n\times750\times10^{-9}\times1}{0.4\times10^{-3}}$

$n=\frac{6\times10^{-6}}{7.5\times10^{-7}}=\frac{60}{7.5}=8$ minima

58.(209) $I=I_0\cos^2\left(\frac{\phi}{2}\right)$, here $I=\frac{I_0}{4}$

$\cos\left(\frac{\phi}{2}\right)=\frac{1}{2} \Rightarrow \frac{\phi}{2}=\frac{\pi}{3} \Rightarrow \phi=\frac{2\pi}{3}$

$\frac{\Delta L}{\lambda}\times2\pi=\frac{2\pi}{3} \Rightarrow \Delta L=\frac{627}{3}nm$; $\Delta L=209\,nm$

59.(4) $(\mu_1-1)t_1=10\lambda \Rightarrow \mu=\frac{3}{2}$ now $(\mu_1-1)t_1=(\mu_2-1)t_2$ $\Rightarrow t_2=4\,\mu m$

60.(3) $I=4I_0\cos^2\frac{\phi}{2}$

Case 1 : $\phi=0 \Rightarrow I=4I_0$

Case 2 : $I=\frac{3I}{4}=4I_0\cos^2\frac{\phi}{2}$ $\Rightarrow \cos\frac{\phi}{2}=\frac{\pm\sqrt{3}}{2} \Rightarrow \frac{\phi}{2}=\frac{\pi}{6}\,;\ \phi=\frac{\pi}{3}$

Now, $\phi=\frac{(\mu-1)t\times2\pi}{\lambda}\,;\ \frac{\pi}{3}=\frac{(\mu-1)t\times2\pi}{\lambda}\,;\ t=\frac{\lambda}{6(\mu-1)}\,;\ t=\frac{6000}{3}=2000\,\text{Å}$

MODERN PHYSICS

1.(C) Energy released = (80 × 7 + 120 × 8 – 200 × 6.5) = 220 MeV

2.(A) $\Delta E = 13.6Z^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$ $\Rightarrow$ $27.2 = 13.6Z^2\left[\frac{1}{4}-\frac{1}{36}\right]$ $\Rightarrow$ $4.25 + x = 13.6Z^2\left[\frac{1}{9}-\frac{1}{36}\right]$

By dividing we get, $\frac{27.2}{4.25+x} = \frac{4}{18}\times 12 = \frac{8}{3}$ $\quad x = 5.95eV$

3.(A) Maximum energy is liberated when transition is from $n = 5$ to $n = 1$ and minimum energy is liberated when transition is from $n = 5$ to $n = 4$.

$$\frac{E_1}{5^2} - E_1 = 52.224 \Rightarrow E_1 = (-)54.4eV \qquad \text{and} \qquad \frac{E_1}{5^2} - \frac{E_1}{4^2} = \frac{9}{400}E_1 = \frac{9}{400}\times 54.4 = 1.224eV$$

4.(A) $I = \frac{Q}{t} = \frac{ne}{t}$ or, $n = \frac{It}{e}$

$$= \frac{(3.2\times 10^{-3})(1)}{(1.6\times 10^{-19})} = 2\times 10^{16}$$

5.(A) $P = h/\lambda$ and $K = \frac{P^2}{2m} = \frac{h^2}{2m\lambda^2}$

For X-ray photons, it is also maximum energy

So, $\frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2}$ or, $\lambda_0 = \frac{2m\lambda^2 c}{h}$

6.(A) $13.6\,(3)^2\left[\frac{1}{3^2}-\frac{1}{4^2}\right] = w + 4$

$13.6\,(3)^2\left[\frac{1}{4^2}-\frac{1}{5^2}\right] = w + V$ (w = work function)

Solving $V = 0.85\ eV$.

7.(B) At time t; $\frac{dN}{dt} = t^2 - \lambda N$

$$\Rightarrow \quad \frac{d^2N}{dt^2} = 2t - \lambda\frac{dN}{dt}\left(\frac{d^2N}{dt^2} = 0\right) \quad \Rightarrow \quad 0 = 2t_0 - \lambda\left(t_0^2 - \lambda N_0\right) \quad \Rightarrow \quad N_0 = \frac{\lambda t_0^2 - 2t_0}{\lambda^2}$$

8.(D) $13.6\left(1-\frac{1}{9}\right) = 13.6z^2\left(\frac{1}{9}-\frac{1}{x^2}\right)$ $\Rightarrow$ $z^2\left(\frac{n^2-9}{n^2}\right) = 8$ $\Rightarrow$ $z = 3$ and $n = 9$

9.(A) No. of neutrons $= \frac{100\times 10^6}{3.2\times 10^{-11}}\times 2.5 = 7.8\times 10^{18}$

10.(A) Let the radius of the n^{th} Bohr orbit be r and let the velocity of the electron in this orbit be v

Angular momentum of the electron, $L = mvr = \frac{nh}{2\pi}$

Also, $\frac{mv^2}{r} = \frac{KZe^2}{r^2}$

Solving, we get $v = \frac{2\pi KZe^2}{nh}$

So, de Broglie wavelength of the electron, $\lambda_n = \frac{h}{mv} = \frac{nh^2}{2\pi KZme^2}$

For the first excited state in the Hydrogen atom, $Z = 1$ and $n = 2$, $\Rightarrow$ $\lambda = \frac{h^2}{\pi KZme^2}$

11.(BC) $K \le 20.4$ eV $\Rightarrow$ no excitation of hydrogen atom $\therefore$ collision will be elastic

12.(AB) $i_m \propto$ intensity, $eV_s = (h\upsilon - \phi_o)$

13.(ABC) $K_{\max} = E - W$

$T_A = 4.25 - W_A$

Given $T_B = T_A - 1.50 = 4.70 - W_B$

On solving, $W_B - W_A = 1.95\, eV$

$$\lambda \propto \frac{1}{K} \quad \left(\text{as } \lambda = \frac{h}{\sqrt{2Km}}\right)$$

$$\frac{\lambda_B}{\lambda_A} = \sqrt{\frac{K_A}{K_B}} \quad ; \quad 2 = \left(\frac{T_A}{T_A - 1.5}\right)^{1/2}$$

On solving, $T_A = 2eV$

$$W_A = 4.25 - T_A = 2.25\, eV$$

$$W_B = W_A + 1.95 = 4.20\, eV$$

$$T_B = 4.70 - W_B = 0.50\, eV$$

14.(AB) $|F| = \frac{dU}{dr} = \frac{Ke^2}{r^4}$ ...... (1) ; $\frac{Ke^2}{r^4} = \frac{mv^2}{r}$ ...... (2) ; $mvr = \frac{nh}{2\pi}$ ...... (3)

Solving (2) and (3) : T.E = KE + PE

Total energy $\propto n^6$; Total energy $\propto m^{-3}$.

15.(ACD) $P.E. = 2K.E.$ and $r_n \propto \frac{n^2}{2}$

16.(ABC) Two or more lighter nuclei combine to form a heavier nucleus in fusion reaction.

17.(BC)

18.(ACD) $c = \frac{E_1}{E_2} = \frac{5R_H/36}{3R_H/4} = \frac{5}{27}$

$$a = \frac{\lambda_1}{\lambda_2} = \frac{1}{c}, b = \frac{1}{a}$$

19.(ACD)

20.(ABCD) Under normal conditions total energy, potential energy and kinetic energy in ground state and first excited state are –13.6 eV,–27.2 eV, 13.6 eV, –3.4 eV, –6.8 eV and 3.4 eV respectively. If potential energy in ground state is taken to be zero, then kinetic energy will remain unchanged but potential and total energies are increased by 27.2 eV. Therefore, the new values are 13.6 eV, 0, 13.6 eV, 23.8 eV and 3.4 eV respectively.

21.(AD) Two or more lighter nuclei are combined to form a relatively heavy nucleus to release the energy

22.(AC) $R = R_0 A^{\frac{1}{3}}$

For O^{16}, $R = R_0 (16)^{\frac{1}{3}}$

For ${}_{54}X^{128}, R' = R_0 (128)^{1/3}$; $R' = \left(\frac{128}{16}\right)^{\frac{1}{3}} R = 2R$; $V' = \frac{4}{3}\pi R^3 = 8V$

23.(BC) (1) Due to emission of $\beta-$ particles mass will almost remain unchanged.

(2) No. of $\beta-$ particles decayed $= 3\times 10^{22}$, so charge $= 3\times 10^{22} \times 1.6\times 10^{-19} = 4800\,C$.

24.(ABCD)

(A) Maximum potential will be equal to the stopping potential which depends on λ and nature of material.

(B) $V = \frac{KQ}{R} \Rightarrow Q = \frac{RV}{K}$

Since V and K are constant, maximum positive charge appearing depends on R.

(C) As the sphere gets charged (which goes on increasing), it applies a force on the emanating electrons thus reduces the velocity of emanating electrons.

(D) Initially the sphere in uncharged, thus $KE_{\max}$ of emanating electron is independent of radius of sphere.

25.(BC) Energy of incident photons, $E = \frac{hc}{\lambda} = \frac{1242}{207} = 6$ eV

Cut-off potential is given, $V_C = 4$ V

Therefore, kinetic energy of the fasting moving photoelectron, $K_{\max} = eV_C = 4$ eV

Work function of plate A, $\phi_0 = E - K_{\max} = 2$ eV

Therefore, longest wavelength of light that can cause emission from plate A,

$$\lambda_m = \frac{hc}{\phi_0} = \frac{1242}{2} = 621 \text{ nm}$$

Number of photons striking plate A per second,

$$N_{ph} = \frac{\text{Energy received at the plate per second}}{\text{Energy per photon}} = \frac{(60)\left(20\times 10^{-4}\right)\left(\sin 30^\circ\right)}{(6)(e)} = \frac{1}{100e}$$

Number of electrons emitted from plate A per second,

$$N_e = \frac{\text{Photoelectric current}}{e} = \frac{5\times 10^{-4}}{e}$$

Therefore one electron is emitted per $\frac{N_{ph}}{N_e}$ photons, i.e., 20 photons.

26.(ACD) $K = \frac{1}{2}mv^2$, $r = \frac{mv}{eB}$, $\lambda = \frac{h}{mv}$

27.(ABC) Radius of the orbit is proportional to n^2 $\Rightarrow$ $n = 4$

Ionization energy $= 13.6\left(\frac{1}{4^2}\right) = 0.85$ eV

$$\lambda = \frac{hc}{13.6\left(\frac{1}{1^2} - \frac{1}{4^2}\right)} = \frac{1242}{13.6}\left(\frac{16}{15}\right) = 97.4 \text{ nm}$$

$$\Delta p_a = \text{Momentum of photon} = \frac{\text{Energy of photon}}{\text{Speed of light}} = \frac{h}{\lambda} = \frac{6.6\times10^{-34}}{97.4\times10^{-9}} = 6.78\times10^{-27} \text{ kg m/s}$$

As the electron can at the most lose as much energy as it gained in the first transition, it can only emit a photon of wavelength higher than 97.4 nm.

28.(BC)

(1) is a β^- decay process, so the mass that converts to energy is just the mass of the parent nucleus minus the mass of the daughter nucleus.

(2) is a β^- decay process, so the mass that converts to energy is the mass of the parent nucleus minus the mass of the daughter nucleus plus **the mass of two electrons**.

29.(AD)

30.(ACD)

$$\frac{hc}{\lambda_L} = 13.6\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) \quad \Rightarrow \quad \lambda_L \text{ is proportional to } \frac{(n(n-1))^2}{(2n-1)}$$

$$\Delta p_L = \text{Momentum of emitted photon} = \frac{h}{\lambda_L} \Rightarrow \Delta p_L \text{ is proportional to } \frac{(2n-1)}{(n(n-1))^2}$$

$$\frac{hc}{\lambda_M} = 13.6\left(\frac{1}{(n-1)^2}\right) \quad \Rightarrow \quad \lambda_M \text{ is proportional to } (n-1)^2$$

31.(B) Ground state energy (in eV) is E_1.

Given condition, $E_{2n} - E_1 = 204\,eV$

$$E_1\left[\frac{1}{4n^2} - 1\right] = 204\,eV \quad \dots (1) \quad \text{and} \quad E_{2n} - E_n = 40.8\,eV$$

$$E_1\left[\frac{1}{4n^2} - \frac{1}{n^2}\right] = -\frac{3E_1}{4n^2} = 40.8\,eV \quad \dots (2)$$

By dividing equation (1) and (2), we get, $\frac{\left[1 - \frac{1}{4n^2}\right]}{\left[\frac{3}{4n^2}\right]} = 5$

On solving, we get, $n = 2$

32.(C) $E_1 = -\frac{4}{3}n^2(40.8)eV = -217.6eV$ (Put $n = 2$)

33.(B) $E_1 = -(13.6)Z^2 = -217.6eV$ or, $Z = 4$

$$E_{min} = E_{2n} - E_{2n-1} = \frac{E}{4n^2} - \frac{E_1}{(2n-1)^2}$$ (Put $n = 2$)

$$= \frac{E_1}{16} - \frac{E_1}{9} = \frac{-7E_1}{144} = +10.58eV$$

34.(B) Decay constant for the decay of A into X, $\lambda_1 = \frac{\log_e 2}{T_1}$

Decay constant for the decay of A into Y, $\lambda_2 = \frac{\log_e 2}{T_2}$

If the instantaneous number of nuclei A is N, then $\frac{dN}{dt} = -(\lambda_1 N + \lambda_2 N) = -(\lambda_1 + \lambda_2)N$

This means that the effective decay constant when both decay processes are going on simultaneously is $\lambda = \lambda_1 + \lambda_2$

So, effective half-life, $T = \frac{\log_e 2}{\lambda} = \frac{T_1 T_2}{T_1 + T_2}$

35.(A) Let the instantaneous number of nuclei A, X and Y present be N_A, N_X and N_Y.

Then, $\frac{dN_X}{dt} = \lambda_1 N_A$ and $\frac{dN_Y}{dt} = \lambda_2 N_A$

Dividing the two equations, we get $\frac{dN_X}{dN_Y} = \frac{\lambda_1}{\lambda_2}$

$\Rightarrow$ $\int \lambda_2 dN_X = \int \lambda_1 dN_Y$ $\Rightarrow$ $\frac{N_X}{N_Y} = \frac{\lambda_1}{\lambda_2}$ $\Rightarrow$ $\frac{N_X}{N_Y} = \frac{T_2}{T_1}$

36.(B) Maximum energy = Binding energy of products – Binding energy of reactants

$\Rightarrow$ $E_{max} = (41.5)(40) - (8.5)(40) = 1320$ keV

37.(D) Let the velocity of the Ca-40 nucleus and the beta particle (electron) after the disintegration be v_1 and v_2 respectively

Conserving momentum, $(40)v_1 + \left(\frac{1}{1800}\right)(-v_2) = 0$ $\Rightarrow$ $v_1 = \frac{v_2}{72000}$

So, ratio of kinetic energies, $$\frac{K_\beta}{K_{Ca}} = \frac{\frac{1}{2}\left(\frac{1}{1800}\right)v_2^2}{\frac{1}{2}(40)v_1^2} = (72000)^2 = 5.18 \times 10^9$$

38.(A) The energy liberated in the decay of a K-40 nucleus into a Ca-40 nucleus is,

$$Q = \left(m(\text{K-40}) - m(\text{Ca-40})\right)(931.5) \text{ MeV}$$

Here $m(\text{K-40})$ and $m(\text{Ca-40})$ denote the mass of a K-40 **atom** and a Ca-40 **atom** respectively.

Therefore, $m(\text{Ca-40}) = m(\text{K-40}) - \frac{Q}{(931.5)} = 39.9640 - 0.0014 = 39.9626$ u

39. **[A – p ; B – r; C – r ; D – r]**

40. **[A – s ; B – r; C – p; D – q]**

41.(1) $r \propto \dfrac{P}{q}$ $\left(\text{Since, } r = \dfrac{P}{Bq}\right)$

Given $r_\alpha = \dfrac{1}{2} r_e$

$\dfrac{P_\alpha}{2} = \dfrac{1}{2}\left(\dfrac{P_e}{1}\right)$ or, $P_\alpha = P_e$

$\lambda \propto \dfrac{1}{P}$ $\left(\text{Since, } \lambda = \dfrac{h}{p}\right)$

So, $\lambda_\alpha = \lambda_e$ or, $n = 1$

42.(6) Given $\lambda_A N_A = \lambda_B N_B$

$$\left(\frac{\ln 2}{T_A}\right)(4N_0 e^{-\lambda_A t}) = (N_0)\left(\frac{\ln 2}{T_B}\right)(e^{-\lambda_B t})$$

$$e^{(\lambda_A - \lambda_B)t} = 8$$

$$(\lambda_A - \lambda_B)t = \ln 8 = 3(\ln 2)$$

$$\left(\frac{\ln 2}{1} - \frac{\ln 2}{2}\right)t = 3\ln(2)$$

43.(7) x and y are number of α- decays and β- decays respectively

$$92 - 2x + y = 85$$

or, $2x - y = 7$ … (1)

Similarly, $238 - 4x = 210$ … (2)

$x = 7$

44.(4) $E_{\text{photon}} = 13.6\left(1 - \dfrac{1}{25}\right)eV = 13.0\,eV$

$E / c = mv$ (Momentum conserved)

$$v = \frac{E}{mc} = \frac{(13)(1.6 \times 10^{-19})}{(1.67)(10^{-27})(3)(10^8)} = 4 \text{ m/sec.}$$

45.(6) $\dfrac{\lambda_1}{\lambda_2} = \dfrac{(Z_2 - 1)^2}{(Z_1 - 1)^2}$ $\left[\text{Since, } \dfrac{1}{\lambda} \propto (Z-1)^2\right]$

$$\frac{1}{4} = \frac{(Z_2 - 1)^2}{(11-1)^2}$$

On solving $Z_2 = 6$

46.(6) When electron jumps from n^{th} state to ground state, number of possible emission lines $= \dfrac{n(n-1)}{2}$.

Here, number of possible emission lines $= \dfrac{(n-1)(n-2)}{2} = 10$ (given)

On solving, $n = 6$

47.(8) $a = v^2 / r$

So, $a \propto \dfrac{Z^2}{(1/Z)}$

Thus, $a \propto Z^3$

$$\frac{a_1}{a_2} = \left(\frac{Z_1}{Z_2}\right)^3 = \left(\frac{2}{1}\right)^3 = 8$$

48.(2) The shortest wavelength of Brackett series is corresponding to transition of electron between $n_1 = 4$ and $n_2 = \infty$. Similarly, the shortest wavelength of Balmer series is corresponding to transition of electron between $n_1 = 2$ and $n_2 = \infty$.

Thus, we have $(Z^2)\left(\frac{13.6}{16}\right) = \left(\frac{13.6}{4}\right)$ or, $Z = 2$

49.(8) $\lambda = \frac{\ell n2}{3} + \frac{\ell n2}{6} = \frac{\ell n2}{2}\ hr^{-1}$ $N = N_0 e^{\frac{-\ell n2}{2} \cdot 6} = N_0 2^{-3}$ $\Rightarrow$ $\frac{N_0}{N} = 8$

50.(8) $\left|\frac{dN}{dt}\right| = \lambda N = \lambda N_0 e^{-\lambda t}$ $\Rightarrow$ $\ell n\left|\frac{dN}{dt}\right| = \ell n\lambda N_0 - \lambda t$ . . . **(i)**

From graph equation is : $\ell n\left|\frac{dN}{dt}\right| - 4 = -\frac{4-3}{6-4}(t-4)$ $\Rightarrow$ $\ell n\left|\frac{dN}{dt}\right| = -\frac{1}{2}t + 6$ . . . **(ii)**

Comparing (i) and (ii)

$$\lambda = \frac{1}{2} \text{ and } \ell n(\lambda N_0) = 2$$

$$N = \frac{N_0}{P} \quad \Rightarrow \quad P = \frac{N_0}{N} = \frac{N_0}{N_0 e^{-\lambda t}} = e^{+\lambda t} e^{+\frac{1}{2} \times 4.16} = e^{2.08} = 8 \text{ (using log table)}$$

51.(9) Let the velocity of the alpha particle before the collision be u_1

Let the velocity of the alpha particle and the deuterium nucleus after the collision be v_1 and v_2 respectively.

Conserving linear momentum, $4u_1 = 4v_1 + 2v_2$

Newton's experimental law, $v_2 - v_1 = u_1$

Solving, we get $v_1 = \frac{u_1}{3}$ and $v_2 = \frac{4u_1}{3}$

So, $$\frac{K}{K'} = \frac{\frac{1}{2}(4)(u_1)^2}{\frac{1}{2}(4)\left(\frac{u_1}{3}\right)^2} = 9$$

52.(4) $13.6\left(\frac{1}{1^2} - \frac{1}{n^2}\right) > 12.7$ $\Rightarrow$ $1 - \frac{1}{n^2} > \frac{127}{136}$

$\Rightarrow$ $\frac{1}{n^2} < \frac{9}{136}$ $\Rightarrow$ $n^2 > \frac{136}{9}$

$\Rightarrow$ $n^2 > 15.11$ $\Rightarrow$ $n \geq 4$

53.(1) $\text{Force} = \frac{(2)(\text{Power received})(\cos i)}{\text{Speed of light}} = \frac{(2)((3)(0.2)(\cos i))(\cos i)}{3 \times 10^8} = 10^{-9}$ N

54.(1.18) Wavelength of K-alpha line is proportional to $\frac{1}{(Z-1)^2}$

Therefore, $\frac{\lambda_{Cr}}{\lambda_{Fe}} = \left(\frac{25}{23}\right)^2 = 1.18$

55.(25.69) $\lambda = \frac{hc}{(13.6)(4)\left(\frac{1}{1^2} - \frac{1}{3^2}\right)} = \frac{1242}{(13.6)(4)}\left(\frac{9}{8}\right) = 25.69 \text{ nm}$

56.(7) (Mass + energy) of the system will remain conserved. Thus (5 + mass energy of A) + (3 + mass energy of B) = (KE of C + mass energy of C + excitation energy) $[5+3+(35-34.99)\times 930 - 8.3]$MeV = KE of C.

(17.3 – 10.3) MeV = KE of C.

57.(159) Radiation αT^4

So $T_2 = 2T_1$ and by Wein's displacement law $\lambda \propto \frac{1}{T}$

So $\lambda_2 = \frac{\lambda_1}{2} = 3000\,\text{Å}$; by Einstein's photoelectric equation $\frac{hc}{\lambda} = eV_s + \phi$

$f = \frac{hc}{\lambda} - eV_s = \frac{hc}{3000\,\text{Å}} - (13.6eV)1^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 4.14 - 2.55$

$\phi = 1.59eV = 1.59 = \left(\frac{\alpha}{100}\right)$ or $\alpha = 159$

58.(255) Energy of the photon $= \frac{hc}{\lambda} = \frac{1240}{6200} \times 51 = 10.2\,eV$

Since six spectral lines are obtained, thus transition is from ground state to $n = 4$.

Also since the atom is not hydrogen, thus only possible atom is He^+.

[Photon of $\lambda = \frac{6200}{51} nm$ corresponds to transition from 4 to 2]

Thus ΔE in collision = 51eV

For minimum kinetic energy of neutron, collision must be perfectly inelastic.

$\frac{1}{2}\mu v_{rel}^2 = 51eV \quad \Rightarrow \quad \frac{1}{2}\frac{4}{5}mv^2 = 51eV \quad \Rightarrow \quad \frac{1}{2}mv^2 = 63.75\,eV$

[m = mass of neutron, v = velocity of neutron]

59.(4) The given wave is superposition of 3 waves with frequency, $\omega_0,\ \omega + \omega_0$ and $\omega_0 - \omega$; $\omega_{max} = (\omega_0 + \omega)$

$\therefore \quad E_{max} = h\nu_{max} = h\frac{(\omega_0 + \omega)}{2\pi}$; $\quad h\nu_{max} = KE_{max} + \phi \Rightarrow \phi = 4eV$

60.(160) $P = 700 \times 10^3 \times 1.6 \times 10^{-19} \times \frac{dN}{dt}$

$= 10 \times 10^{-3}; \frac{dN}{dt} = \frac{10^{-2}}{10^{-14}} \times \frac{1}{7 \times 16} = \frac{10^{12}}{11.2} = \lambda N_0$; $\quad \lambda = \frac{\ln 2}{14 \times 86400} \Rightarrow N_0 = \frac{14 \times 86400 \times 10^{-12}}{11.2 \ln 2} = 160 \times 10^{15}$